Circuit Analysis and Design

by

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Exercises

Exercise 1-1 Convert the following quantities to scientific notation: (a) 52 mV, (b) 0.3 MV, (c) 136 nA, (d) 0.05 Gbits/s.

Solution:

(a) $52\ \text{mV} = 52 \times 10^{-3}\ \text{V} = 5.2 \times 10^{-2}\ \text{V}$

(b) $0.3\ \text{MV} = 0.3 \times 10^{6}\ \text{V} = 3 \times 10^{5}\ \text{V}$

(c) $136\ \text{nA} = 136 \times 10^{-9}\ \text{A} = 1.36 \times 10^{-7}\ \text{A}$

(d) $0.05\ \text{Gbits/s} = 0.05 \times 10^{9}\ \text{bits/s} = 5 \times 10^{7}\ \text{bits/s}$

Exercise 1-2 Convert the following quantities to a prefix format such that the number preceding the prefix is between 1 and 999: (a) 8.32×10^7 Hz, (b) 1.67×10^{-8} m, (c) 9.79×10^{-16} g, (d) 4.48×10^{13} V, (e) 762 bits/s.

Solution:

(a) 8.32×10^7 Hz $= 83.2 \times 10^6$ Hz $= 83.2$ MHz

(b) 1.67×10^{-8} m $= 16.7 \times 10^{-9}$ m $= 16.7$ nm

(c) 9.79×10^{-16} g $= 979 \times 10^{-18}$ g $= 979$ ag

(d) 4.48×10^{13} V $= 44.8 \times 10^{12}$ V $= 44.8$ TV

(e) 762 bits/s $= 762$ bits/s

Exercise 1-3 Simplify the following operations into a single number, expressed in prefix format: (a) $A = 10\ \mu\text{V} + 2.3\ \text{mV}$, (b) $B = 4\ \text{THz} - 230\ \text{GHz}$, (c) $C = 3\ \text{mm}/60\ \mu\text{m}$.

Solution:

(a) $A = 10\ \mu\text{V} + 2.3\ \text{mV} = 10 \times 10^{-3} \times 10^{-3}\ \text{V} + 2.3\ \text{mV}$
$= 0.01\ \text{mV} + 2.3\ \text{mV} = 2.31\ \text{mV}$

(b) $B = 4\ \text{THz} - 230\ \text{GHz} = 4\ \text{THz} - 230 \times 10^{-3}\ \text{THz}$
$= 4\ \text{THz} - 0.23\ \text{THz} = 3.77\ \text{THz}$

(c) $C = 3\ \text{mm}/60\ \mu\text{m} = 3 \times 10^{-3}\ \text{m}/60 \times 10^{-6}\ \text{m} = 50$

Exercise 1-4 If the current flowing through a given resistor in a circuit is given by $i(t) = 5[1 - e^{-2t}]$ A for $t \geq 0$, determine the total amount of charge that passed through the resistor between $t = 0$ and $t = 0.2$ s.

Solution: Based on Eq. (1.6):

$$\begin{aligned} q(t) &= \int_{-\infty}^{t} i(t)\, dt \\ &= \int_{0}^{0.2} 5[1 - e^{-2t}]\, dt \\ &= 5\left[t - \frac{e^{-2t}}{-2}\right]_{0}^{0.2} \\ &= 5[(0.2 + 0.5e^{-0.4}) - (0 + 0.5e^{0})]\ \text{C} \\ &= 0.18\ \text{C} \end{aligned}$$

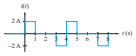
Exercise 1-5 If $q(t)$ has the waveform shown in Fig. E1-5, determine the corresponding current waveform.

Figure E1-5

Solution: Based on Eq. (1.3), $i(t)$ can be calculated and then plotted. First let's express q and i as a function of t:

	q	i
$t < 0$	0	0
$0 \le t < 1$	$2t$	2
$1 \le t < 3$	2	0
$3 \le t < 4$	$8 - 2t$	-2
$4 \le t < 5$	$-8 + 2t$	2
$5 \le t < 7$	2	0
$7 \le t < 8$	$16 - 2t$	-2

$i(t)$ is equal to the slope of $q(t)$.



Exercise 1-6 If a positive current is flowing through a resistor, from its terminal a to its terminal b, is v_{ab} positive or negative?

Solution: If positive current is flowing from terminal a to terminal b of a resistor, then terminal a is at a higher potential than terminal b making $v_{ab} = (v_a - v_b)$ positive.

Exercise 1-7 A certain device has a voltage difference of 5 V across it. If 2 A of current is flowing through it from its (−) voltage terminal to its (+) terminal, is the device a power supplier or a power recipient, and how much energy does it supply or receive in 1 hour?

Solution:

$$I = -2 \text{ A} \qquad \text{(flowing from negative to positive terminals)}$$
$$V = 5 \text{ V}$$
$$P = VI = -10 \text{ W}$$

By passive sign convention, device is a power supplier.

Exercise 1-8 A car radio draws 0.5 A of dc current when connected to a 12-V battery. How long does it take for the radio to consume 1.44 kJ?

Solution:

$$P = IV = 0.5 \times 12 = 6 \text{ W}$$

$$\Delta t = \frac{W}{P} = \frac{1.44}{6} \times 10^3 = 240 \text{ s} = 4 \text{ minutes}.$$

Exercise 1-9 Find I_x from the diagram in Fig. E1-9.

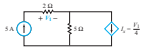


Figure E1-9

Solution:

$$V_1 = 5 \times 2 = 10 \text{ V}$$

$$I_x = \frac{V_1}{4} = 2.5 \text{ A}.$$

Exercise 1-10 In the circuit of Fig. E1-10, find I at (a) $t < 0$ and (b) $t > 0$.

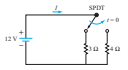


Figure E1-10

Solution:

(a) At $t < 0$,

$$I = \frac{12}{3} = 4 \text{ A}.$$

(b) At $t > 0$,

$$I = \frac{12}{4} = 3 \text{ A}.$$

Exercise 2-1 A cylindrical resistor made of carbon has a power rating of 2 W. If its length is 10 cm and its circular cross section has a diameter of 1 mm, what is the maximum current that can flow through the resistor without damaging it?

Solution: According to Eq. (2.2),

$$R = \frac{\rho \ell}{A}.$$

From Table 2-1, the resistivity of carbon is

$$\rho = 1.4 \times 10^{-5}\ \Omega\text{-m},$$

and the cross-sectional area of the wire is

$$A = \pi \left(\frac{d}{2}\right)^2 = \pi \times (0.5 \times 10^{-3})^2$$
$$= 7.85 \times 10^{-7}\ \text{m}^2.$$

Hence,

$$R = \frac{1.4 \times 10^{-5} \times 10 \times 10^{-2}}{7.85 \times 10^{-7}} = 1.78\ \Omega,$$

and from

$$P_\text{m} = I_\text{m}^2 R = 2\ \text{W},$$
$$I_\text{m} = \sqrt{\frac{2}{R}} = \sqrt{\frac{2}{1.78}} = 1.06\ \text{A}.$$

Exercise 2-2 A rectangular bar made of aluminum has a current of 3 A flowing through it along its length. If its length is 2.5 m and its square cross section has 1-cm sides, how much power is dissipated in the bar at 20°C?

Solution: From Eq. (2.2) and Table 2-1,

$$R = \frac{\rho \ell}{A} = \frac{2.62 \times 10^{-8} \times 2.5}{(10^{-2})^2} = 6.55 \times 10^{-4}\ \Omega,$$

$$P = I^2 R = 3^2 \times 6.55 \times 10^{-4} = 5.9 \times 10^{-3}\ \text{W} = 5.9\ \text{mW}.$$

Exercise 2-3 A certain type of diode exhibits a nonlinear relationship between v, the voltage across it, and i, the current entering into its $(+)$ voltage terminal. Over its operational voltage range (0–1 V), the current is given by

$$i = 0.5v^2, \qquad \text{for } 0 \leq v \leq 1 \text{ V}.$$

Determine how the diode's effective resistance varies with v and calculate its value at $v = 0$, 0.01 V, 0.1 V, 0.5 V, and 1 V.

Solution: The effective resistance of the diode is:

$$R = \frac{v}{i} = \frac{v}{0.5v^2} = \frac{1}{0.5v} = \frac{2}{v}\,.$$

Hence,

v	R
0	∞
0.01 V	200 Ω
0.1 V	20 Ω
0.5 V	4 Ω
1 V	2 Ω

Exercise 2-4 If $I_1 = 3$ A in Fig. E2-4, what is I_2?

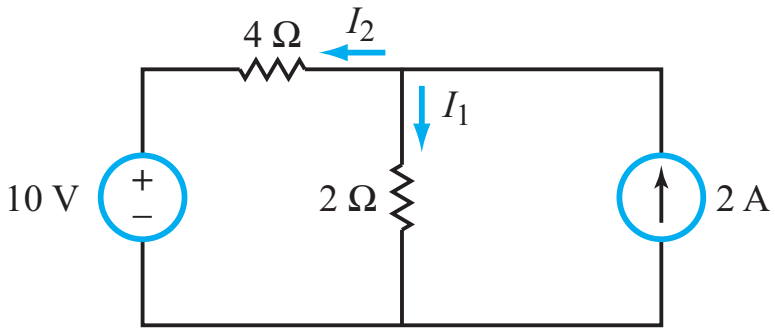


Figure E2-4

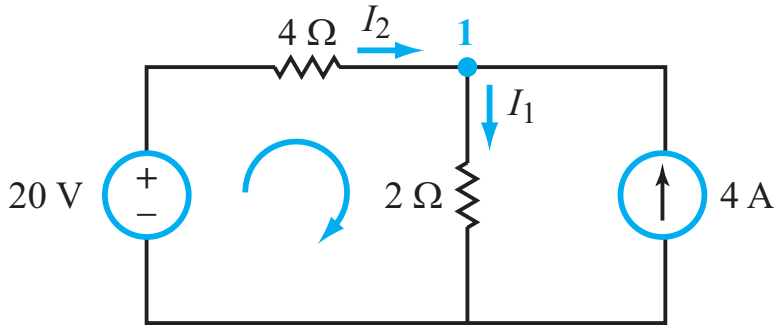
Solution: KCL at the top center node requires that

$$I_1 + I_2 - 2 \text{ A} = 0.$$

Hence,

$$I_2 = 2 - I_1 = 2 - 3 = -1 \text{ A}.$$

Exercise 2-5 Apply KCL and KVL to find I_1 and I_2 in Fig. E2-5.



Solution: KCL at node 1 requires that

$$I_1 = I_2 + 4.$$

Also, KVL for the left loop is

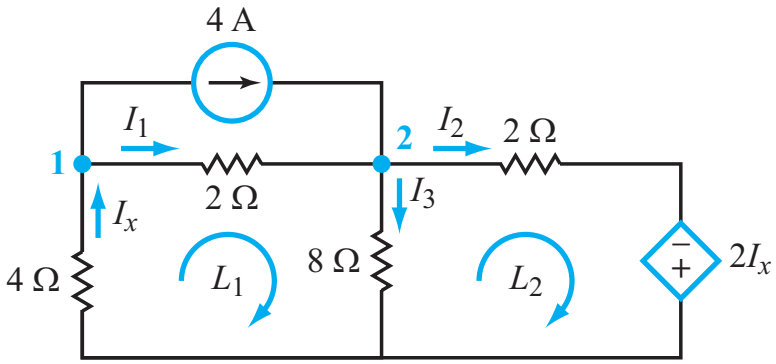
$$-20 + 4I_2 + 2I_1 = 0.$$

Simultaneous solution leads to

$$I_1 = 6 \text{ A}, \qquad I_2 = 2 \text{ A}.$$

Exercise 2-6 Determine I_x in the circuit of Fig. E2-6.

Solution:



KCL @ node 1: $I_x = I_1 + 4$
KCL @ node 2: $I_1 + 4 = I_2 + I_3$
KVL Loop 1: $4I_x + 2I_1 + 8I_3 = 0$
KVL Loop 2: $-8I_3 + 2I_2 - 2I_x = 0$

We have four equations with four unknowns. Simultaneous solution leads to

$$I_x = 1.33.$$

Exercise 2-7 Apply resistance combining to simplify the circuit of Fig. E2-7 so as to find I. All resistor values are in ohms.

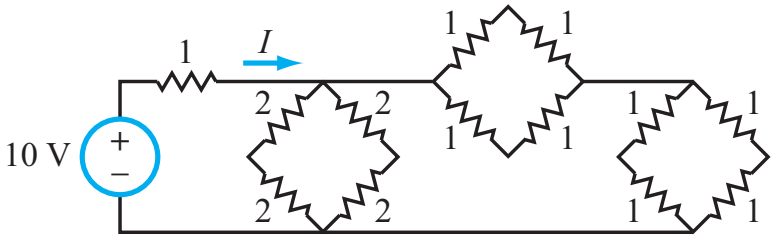
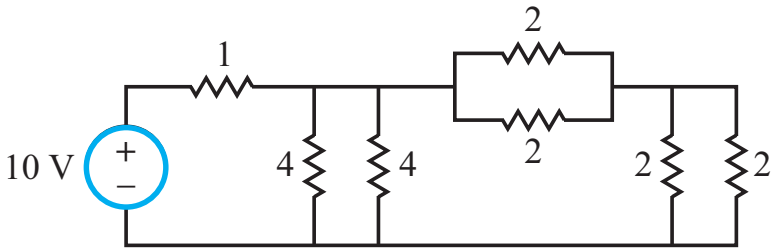
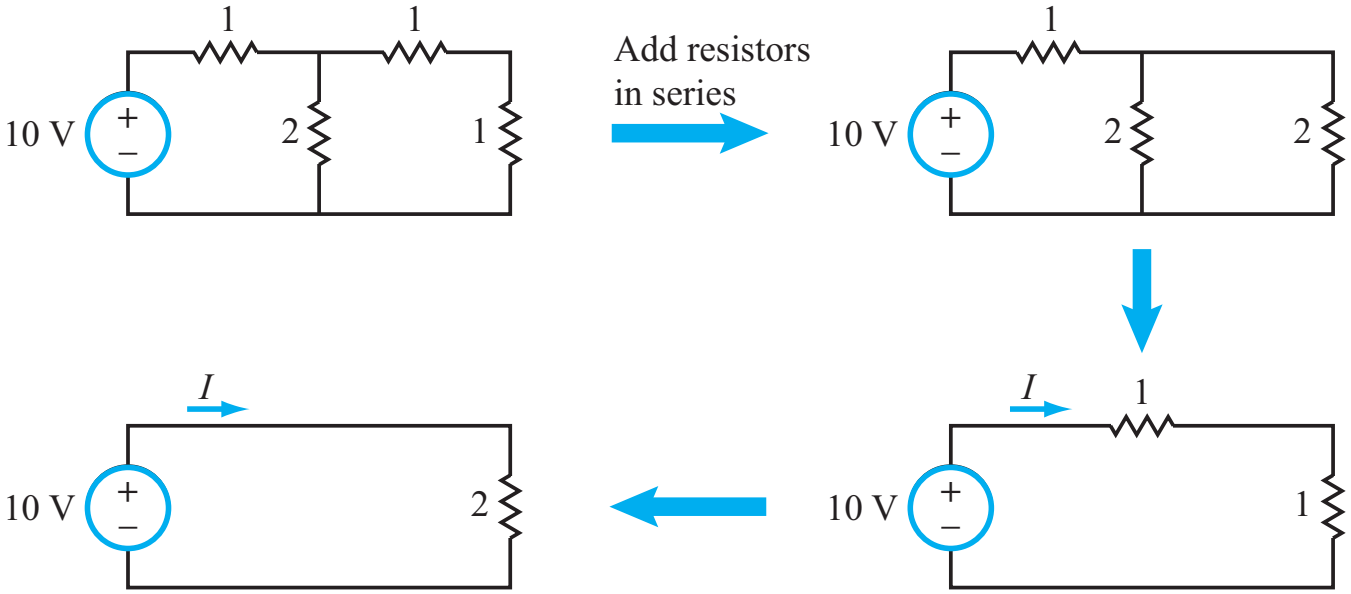


Figure E2-7

Solution: Combining all resistors that are in series will result in the following circuit:



Combining all resistors that are in parallel will result in:



$$I = \frac{10\text{ V}}{2\ \Omega} = 5\text{ A}.$$

Exercise 2-8 Apply source transformation to the circuit in Fig. E2-8 to find I.

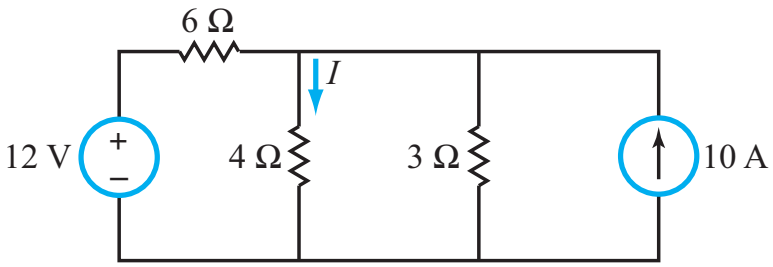
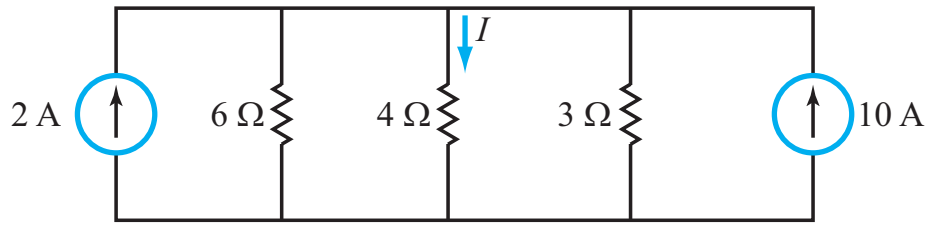
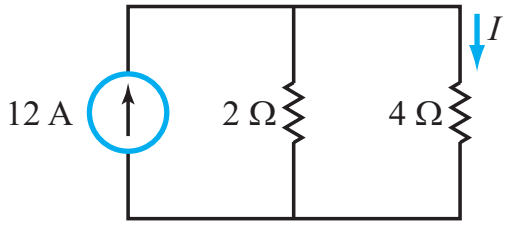


Figure E2-8

Solution: Apply source transformation to the 12-V source and 6-Ω resistor:



Combine current sources and combine 3-Ω and 6-Ω resistors, while leaving 4-Ω alone



Current division gives

$$I = \frac{12 \times 2}{2+4} = 4 \text{ A}.$$

Exercise 2-9 For each of the circuits shown in Fig. E2-9, determine the equivalent resistance between terminals (a,b).

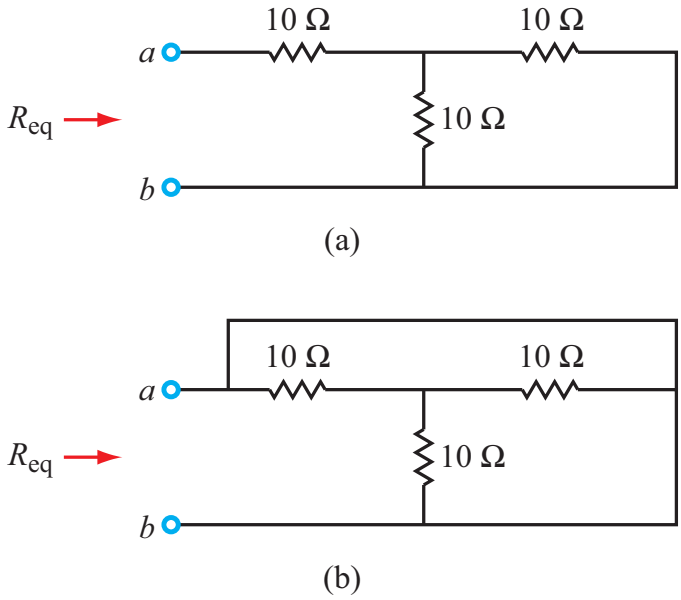
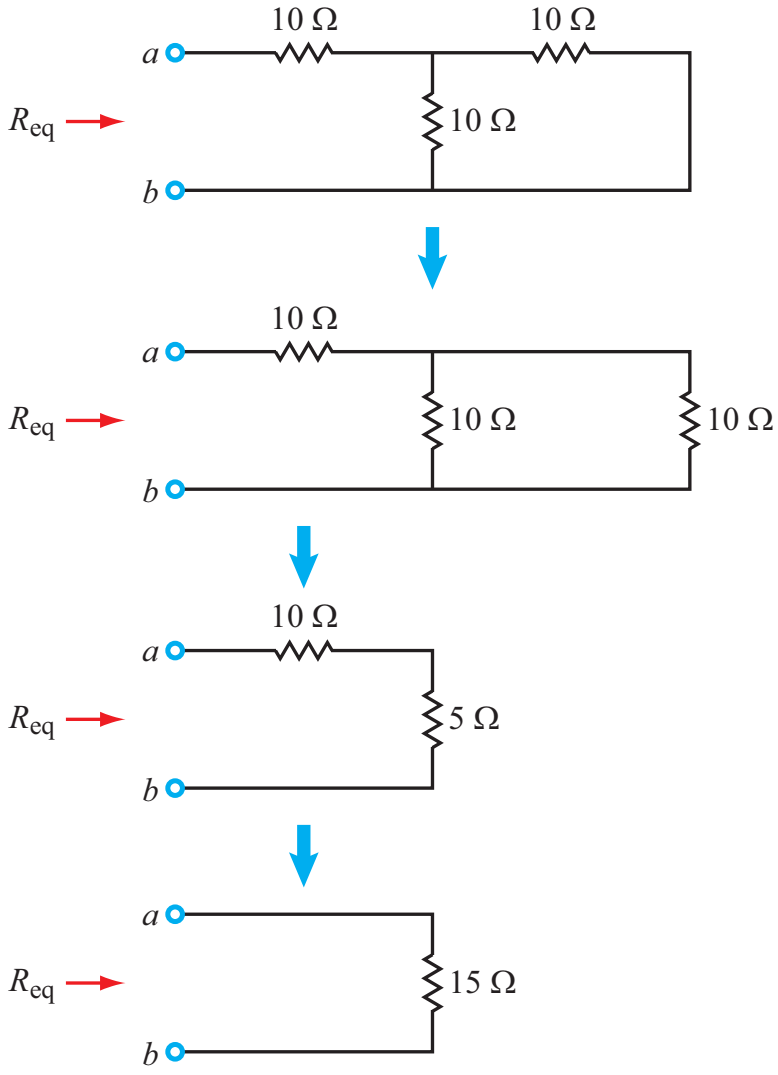


Figure E2-9

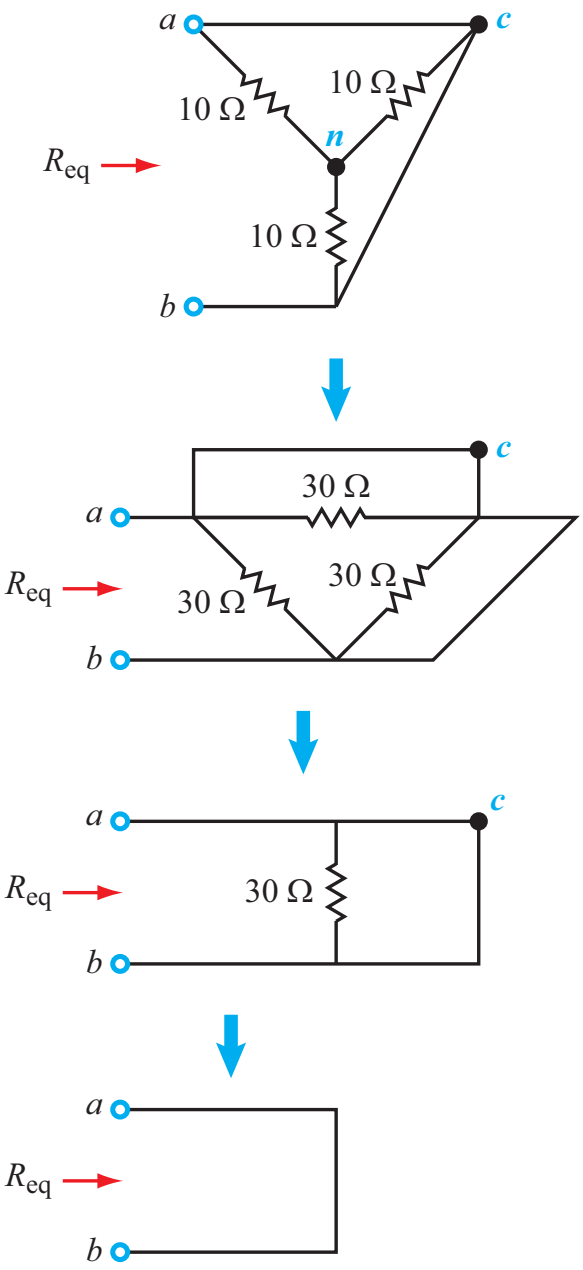
Solution:

(a)



$R_{eq} = 15\ \Omega.$

(b) Applying Y-Δ transformation



$R_{eq} = 0.$

Exercise 2-10 If in the sensor circuit of Fig. 2-37, $V_0 = 4$ V and the smallest value of V_{out} that can be measured reliably is 1 μV, what is the corresponding accuracy with which $(\Delta R/R)$ can be measured?

Solution:

$$V_{\text{out}} = \frac{V_0}{4}\left(\frac{\Delta R}{R}\right),$$

$$\frac{\Delta R}{R} = \frac{4V_{\text{out}}}{V_0} = \frac{4 \times 10^{-6}}{4} = 10^{-6}$$

Exercise 2-11 Determine I in the two circuits of Fig. E2-11. Assume $V_F = 0.7$ V for all diodes.

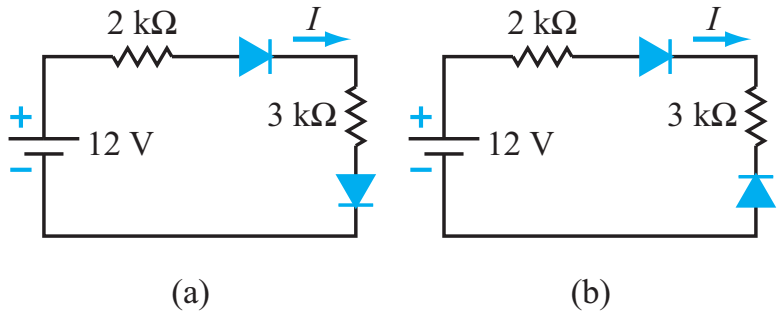


Figure E2-11

Solution:

(a) With $V_F = 0.7$ V, KVL around the loop gives

$$-12 + 2 \times 10^3 I + 0.7 + 3 \times 10^3 I + 0.7 = 0,$$

which leads to

$$I = \frac{12 - 1.4}{5 \times 10^3} = 2.12 \text{ mA}.$$

(b) Since the diodes are biased in opposition to one another, no current can flow in the circuit. Hence

$$I = 0.$$

Exercise 2-12 What would the output voltage associated with the circuit of Example 2-18 change to, if the cantilever thickness is reduced by a factor of 2?

Solution:

$$V_{\text{out}} \sim \frac{1}{H^2}\,.$$

Hence V_{out} will change to

$$V_{\text{out}}(\text{new}) = \frac{V_{\text{out}}(\text{old})}{H^2(\text{new})}\,H^2(\text{old}) = \frac{-0.1}{(0.5)^2} = -0.4\ \text{V}.$$

Exercise 2-13 The circuit in Fig. E2-13 is called a resistive bridge. How does $V_x = (V_3 - V_2)$ vary with the value of potentiometer R_1?

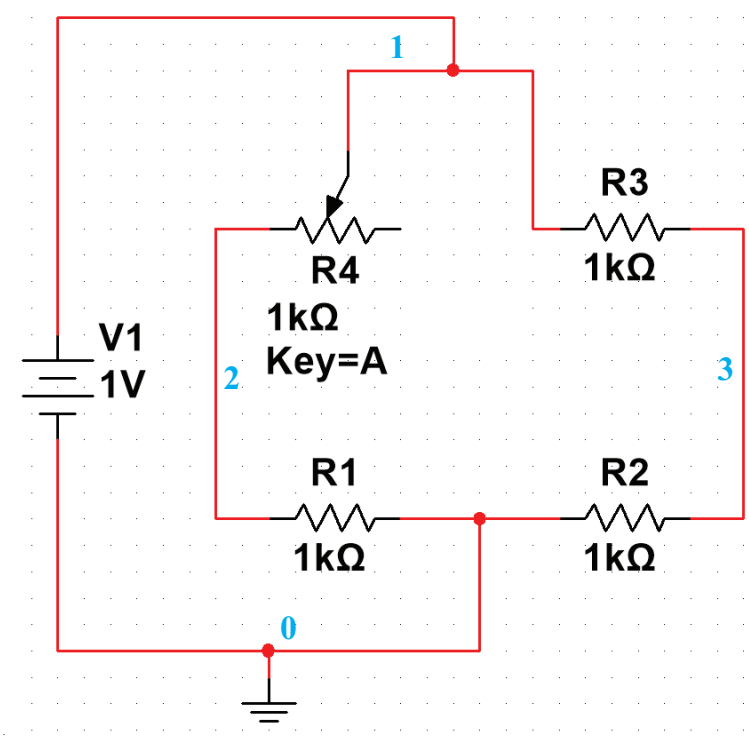


Figure E2-13

Solution: Using DC Operating Point Analysis and varying the value of the potentiometer, we obtain the following values for $(V_3 - V_2)$:

R1 (% of 1 kΩ)	$V_x = V_2 - V_3$
100%	0 mV
80%	55.6 mV
60%	125 mV
40%	214 mV
20%	333 mV
0%	500 mV

Exercise 2-14 Simulate the circuit shown in Fig. E2-14 and solve it for the voltage across R_3. The magnitude of the dependent current source is $V_1/100$.

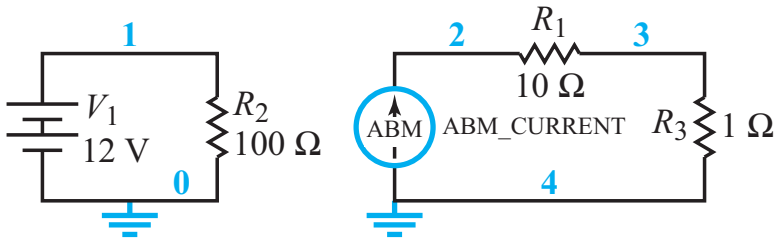
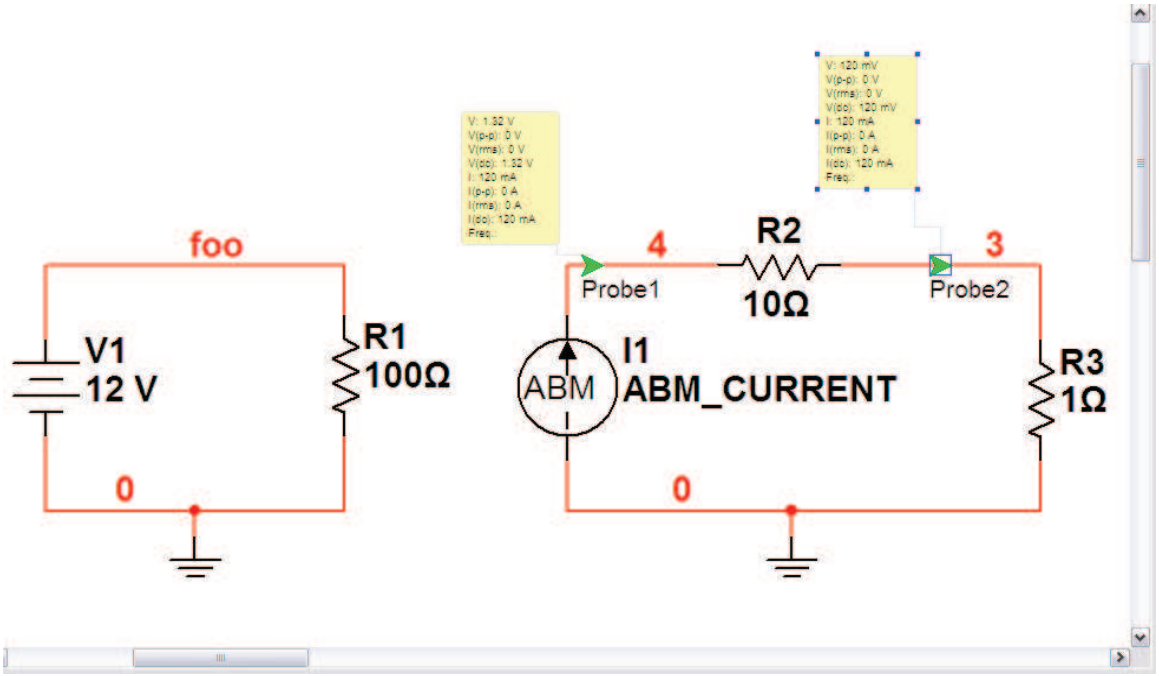


Figure E2-14

Solution: The circuit drawn in Multisim is shown below.



Note: The expression entered into the ABM_CURRENT source was V(foo)/100. The node above the source V1 was renamed from "1" to "foo" (by double-clicking on the wire) to avoid confusing the expression for the source voltage, vv(1), and the symbol for the node voltage, V(1). Once renamed, the symbol for the node becomes V(foo) and the ABM_CURRENT expression becomes V(foo)/100. Had we left the node with the name "1", the ABM_CURRENT expression would have been V(1)/100.

The Measurement Probe on node 3 shows us that the voltage on R_3, relative to ground, is 120 mV.

Exercise 3-1 Apply nodal analysis to determine the current I.

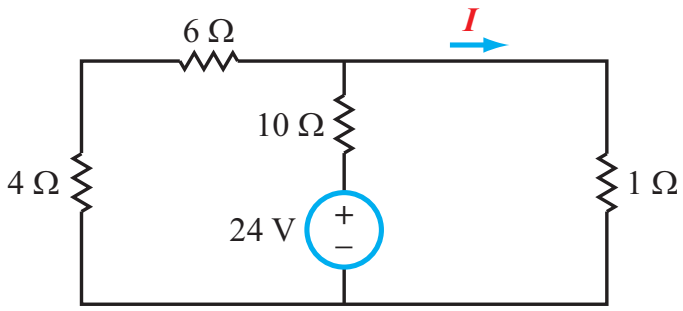
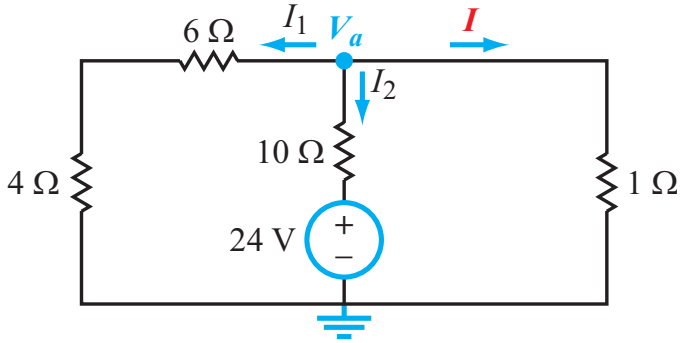


Figure E3-1

Solution:



$$I_1 + I_2 + I = 0$$

$$I_1 = \frac{V_a}{10}, \qquad I_2 = \frac{V_a - 24}{10}, \qquad I_3 = \frac{V_a}{1}$$

Hence,

$$\frac{V_a}{10} + \frac{V_a - 24}{10} + V_a = 0,$$

$$V_a\left(\frac{1}{10} + \frac{1}{10} + 1\right) = \frac{24}{10},$$

which leads to

$$V_a = 2 \text{ V}, \qquad I = \frac{V_a}{1} = 2 \text{ A}.$$

Exercise 3-2 Apply nodal analysis to find V_a.

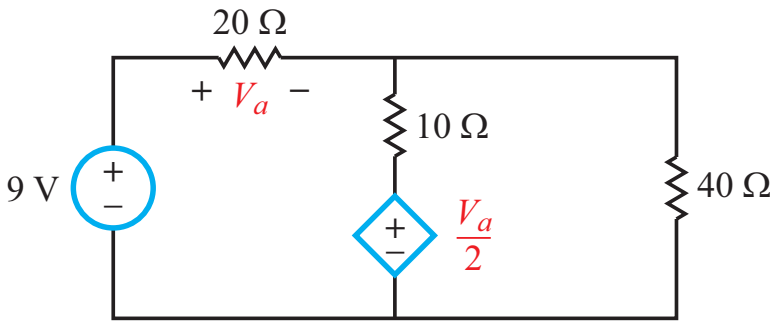
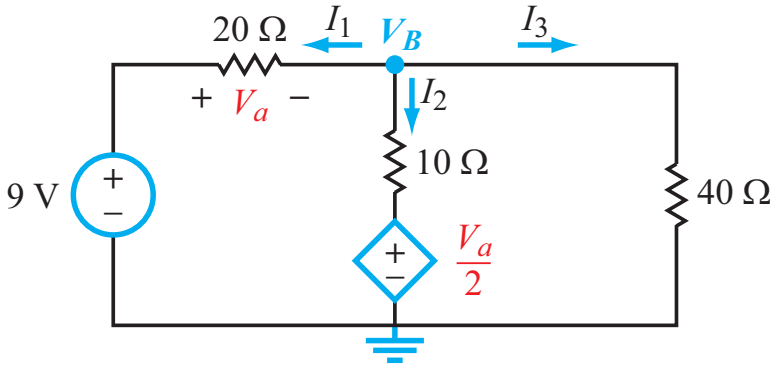


Figure E3-2

Solution:



$$I_1 + I_2 + I_3 = 0$$

$$I_1 = \frac{V_B - 9}{20}, \qquad I_2 = \frac{V_B - \frac{V_a}{2}}{10}, \qquad I_3 = \frac{V_B}{40}.$$

Hence,

$$\frac{V_B - 9}{20} + \frac{V_B - \frac{V_a}{2}}{10} + \frac{V_B}{40} = 0.$$

Also,

$$V_A = 9 - V_B.$$

Solution gives: $V_a = 5$ V.

Exercise 3-3 Apply the supernode concept to determine I in the circuit of Fig. E3-3.

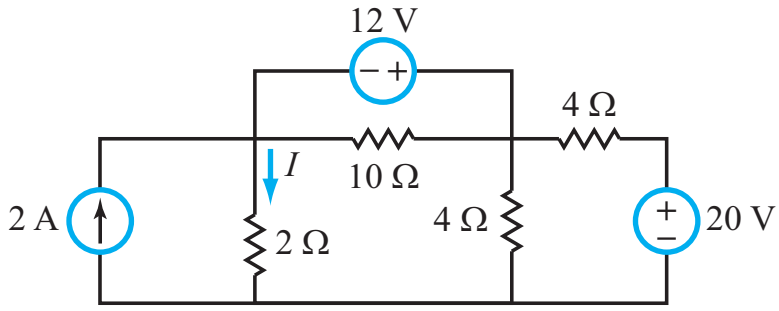
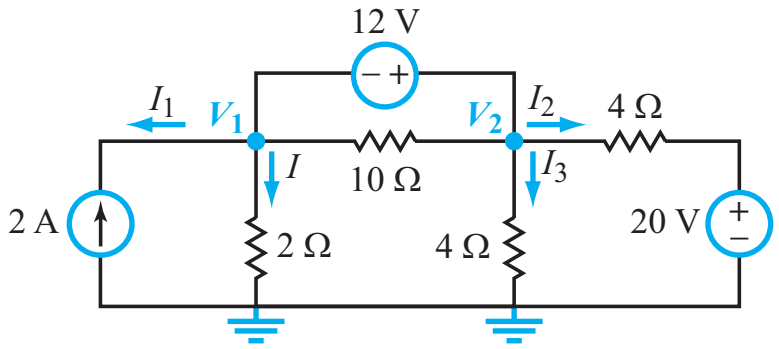


Figure E3.3

Solution:



(V_1, V_2) constitutes a supernode. Hence,

$$I_1 + I + I_2 + I_3 = 0,$$

$$I_1 = -2 \text{ A}, \qquad I = \frac{V_1}{2},$$

$$I_3 = \frac{V_2}{4}, \qquad I_2 = \frac{V_2 - 20}{4}.$$

Also,

$$V_2 - V_1 = 12.$$

Solution leads to: $I = 0.5$ A.

Exercise 3-4 Apply mesh analysis to determine I.

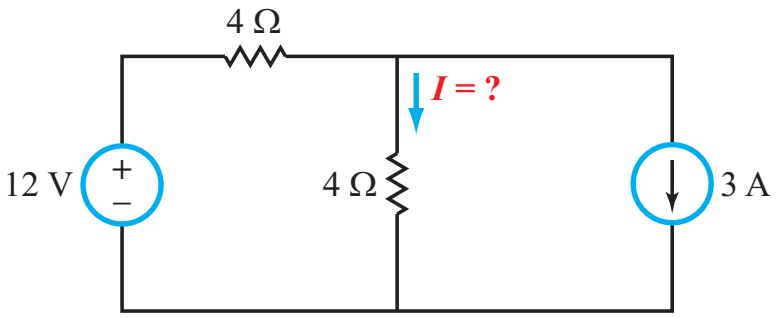
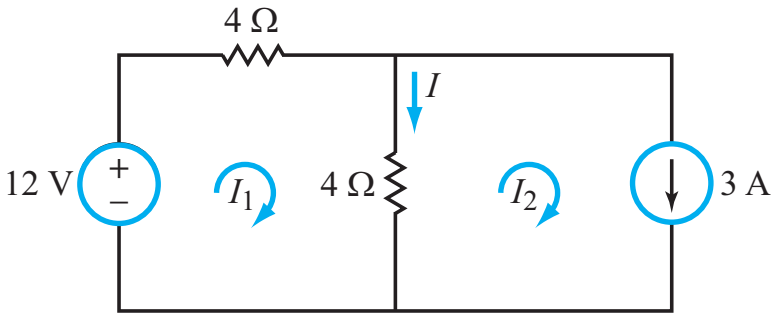


Figure E3-4

Solution:



Mesh 1: $-12+4I_1+4(I_1-I_2)=0$

Mesh 2: $I_2=3$ A

$$4I_1+4I_1-4\times 3=12$$

$$8I_1=24$$

$$I_1=3 \text{ A}.$$

$$\Longrightarrow \quad I=I_1-I_2=3-3=0.$$

Exercise 3-5 Determine the current I in the circuit of Fig. E3-5.

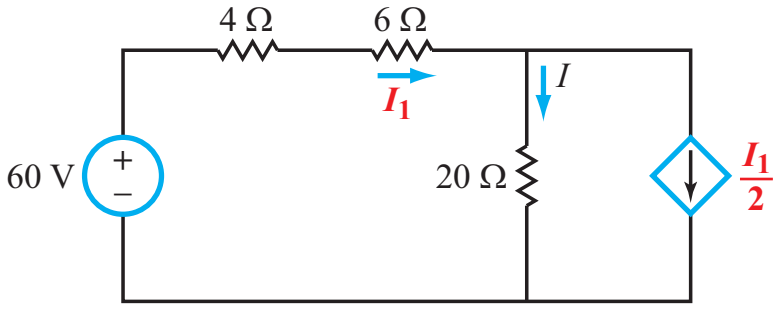
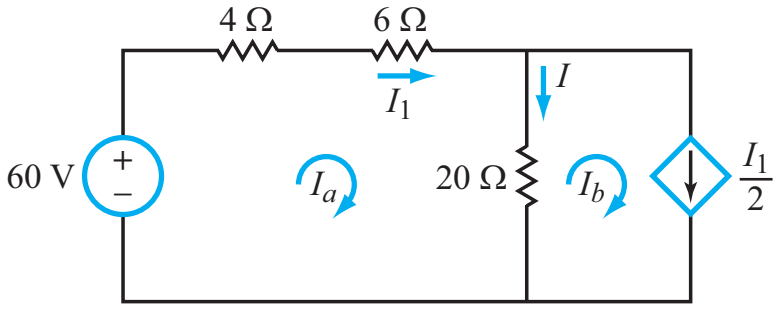


Figure E3-5

Solution:



Mesh 1: $-60+10I_a+20(I_a-I_b)=0$

Mesh 2: $I_b=\dfrac{I_1}{2}$

Also,

$$I_1=I_a.$$

Hence,

$$I_b=\frac{I_a}{2},$$

$$-60+10I_a+20\left(I_a-\frac{I_a}{2}\right)=0,$$

which simplifies to

$$20I_1=60$$

or

$$I_a=3\text{ A},$$

$$I=I_a-I_b=I_a-\frac{I_a}{2}=\frac{I_a}{2}=\frac{3}{2}=1.5\text{ A}.$$

Exercise 3-6 Apply mesh analysis to determine I in the circuit of Fig. E3-6.

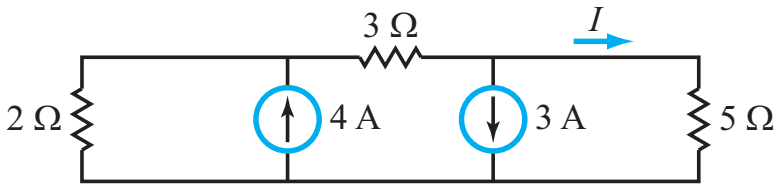
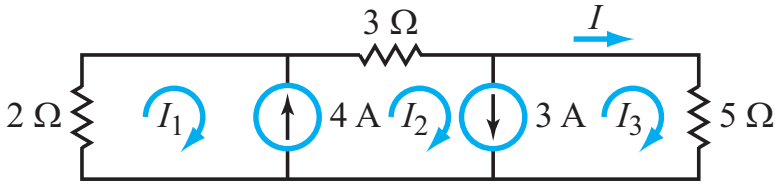


Figure E3-6

Solution:



Outside mesh: $2I_1 + 3I_2 + 5I_3 = 0$.

Also,

$$I_2 - I_1 = 4 \text{ A}, \qquad I_2 - I_3 = 3 \text{ A}.$$

Hence,

$$I_1 = I_2 - 4 = (I_3 + 3) - 4 = I_3 - 1$$
$$I_2 = I_3 + 3$$

$$2(I_3 - 1) + 3(I_3 + 3) + 5I_3 = 0$$
$$10I_3 = 2 - 9$$
$$I_3 = -0.7 \text{ A}$$

$$I = I_3 = -0.7 \text{ A}.$$

Exercise 3-7 Apply the node-analysis by-inspection method to generate the node voltage matrix for the circuit in Fig. E3-7.

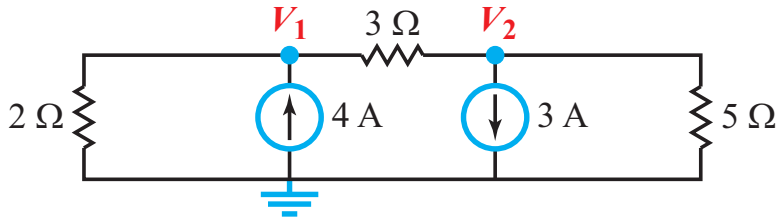


Figure E3.7

Solution:

$$G_{11} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}, \qquad G_{22} = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}, \qquad G_{11} = \frac{5}{6},$$

$$G_{12} = -\frac{1}{3} = G_{21}, \qquad G_{22} = \frac{8}{15}.$$

Hence,

$$\begin{bmatrix} \frac{5}{6} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{8}{15} \end{bmatrix} \begin{bmatrix} V_1 \\ \\ V_2 \end{bmatrix} = \begin{bmatrix} 4 \\ \\ -3 \end{bmatrix}.$$

By MATLAB software,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3.4 \text{ V} \\ -3.5 \text{ V} \end{bmatrix}.$$

Exercise 3-8 Use the by-inspection method to generate the mesh current matrix for the circuit in Fig. E3-8.

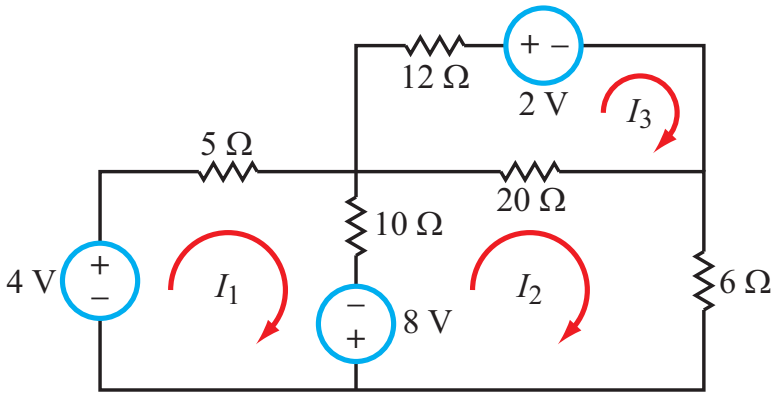


Figure E3-8

Solution:

$$R_{11} = 5 + 10 = 15$$
$$R_{22} = 10 + 20 + 6 = 36$$
$$R_{33} = 20 + 12 = 32$$
$$R_{12} = R_{21} = -10$$
$$R_{13} = R_{31} = 0$$
$$R_{23} = R_{32} = -20$$

Hence,

$$\mathbf{R} = \begin{bmatrix} 15 & -10 & 0 \\ -10 & 36 & -20 \\ 0 & -20 & 32 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 8 + 4 = 12 \\ -8 \\ -2 \end{bmatrix}$$

$$\mathbf{I} = \mathbf{R}^{-1}\mathbf{V} = \begin{bmatrix} 0.7505 \\ -0.0743 \\ -0.1089 \end{bmatrix}$$

$$\therefore \quad I_1 = 0.75 \text{ A}$$
$$I_2 = -0.07 \text{ A}$$
$$I_3 = -0.11 \text{ A}$$

Exercise 3-9 Apply the source-superposition method to determine the current I in the circuit of Fig. E3-9.

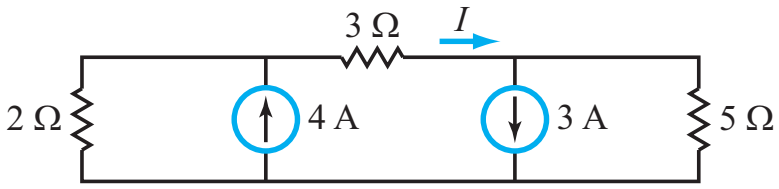
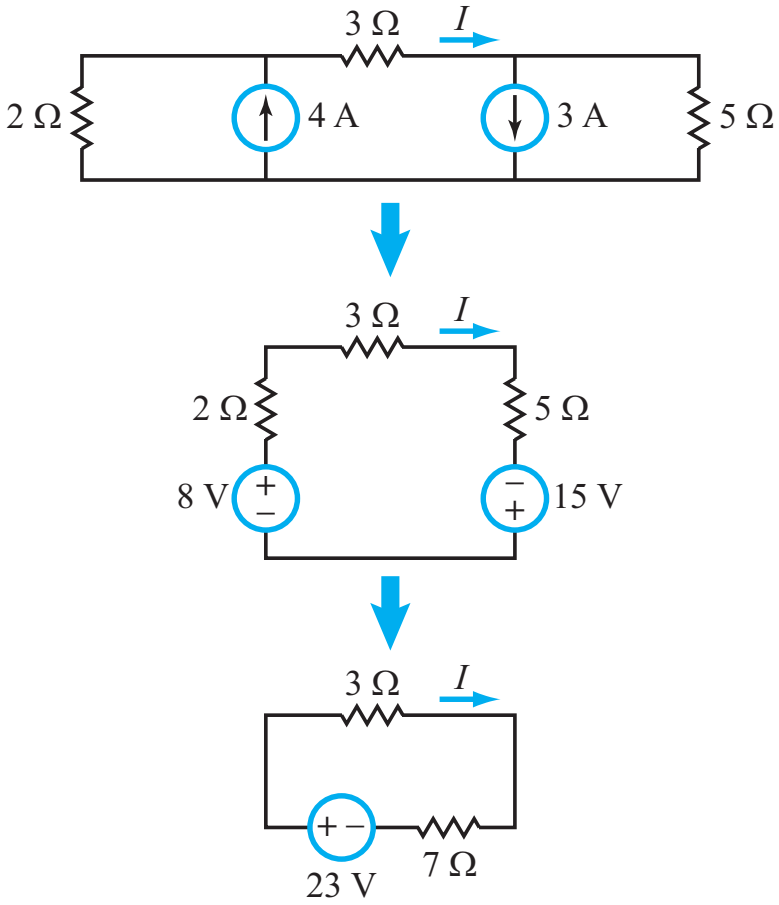


Figure E3-9

Solution:



$$I = \frac{23}{3+7} = 2.3 \text{ A}.$$

Exercise 3-10 Apply source superposition to determine V_{out} in the circuit of Fig. E3-10.

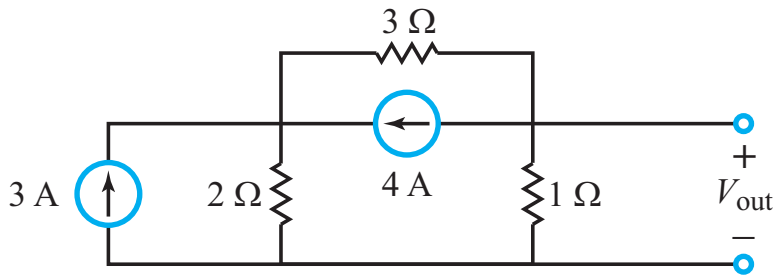
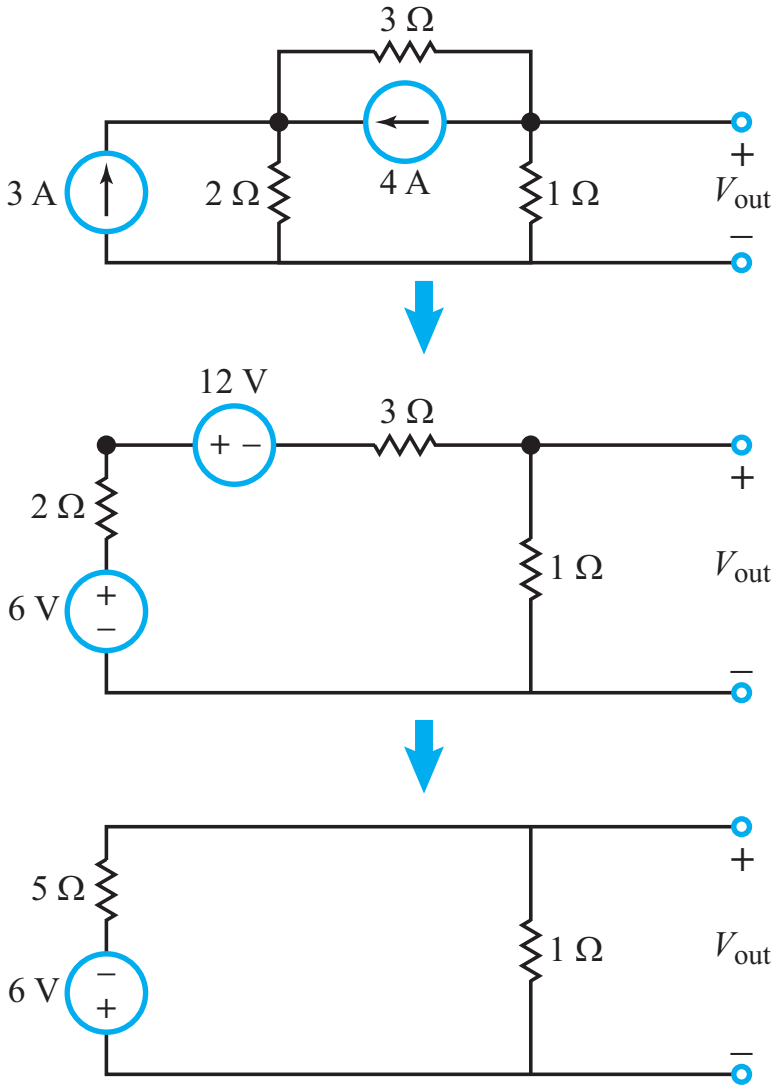


Figure E3-10

Solution:



By voltage division,

$$V_{out} = \frac{-6 \times 1}{5+1} = -1 \text{ V}.$$

Exercise 3-11 Determine the Thévenin-equivalent circuit at terminals (a,b) in Fig. E3-11.

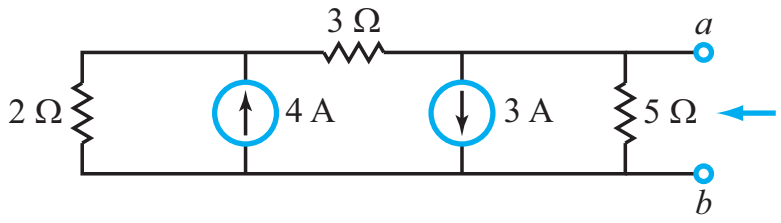
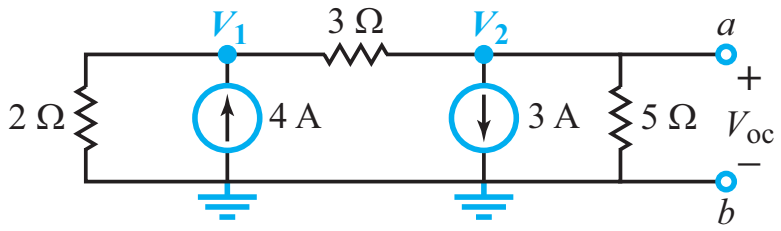


Figure E3.11

Solution:

(1) Open-circuit voltage

We apply node voltage method to determine open-circuit voltage:



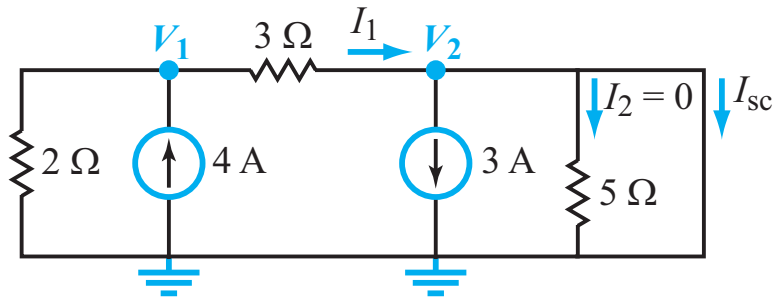
$$\frac{V_1}{2} - 4 + \frac{V_1 - V_2}{3} = 0,$$
$$\frac{V_2 - V_1}{3} + 3 + \frac{V_2}{5} = 0.$$

Solution gives: $V_2 = -3.5$ V.

Hence,

$$V_{\text{Th}} = V_{\text{oc}} = -3.5 \text{ V}.$$

(2) Short-circuit current



Because of the short circuit,

$$V_2 = 0.$$

Hence at node V_1:

$$\frac{V_1}{2} - 4 + \frac{V_1}{3} = 0$$

$$V_1\left(\frac{1}{2} + \frac{1}{3}\right) = 4$$

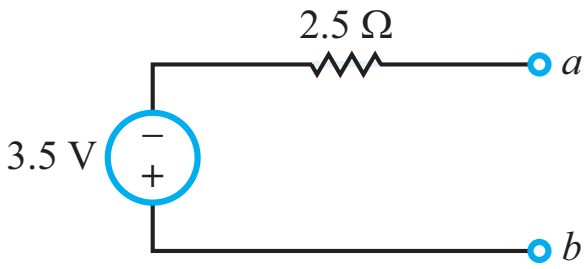
$$V_1 = \frac{24}{5}\ \text{V}$$

$$I_1 = \frac{V_1}{3} = \frac{24}{5 \times 3} = \frac{8}{5}\ \text{A},$$

$$I_{sc} = I_1 - 3 = \frac{8}{5} - 3 = -\frac{7}{5} = -1.4\ \text{A}$$

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{-3.5}{-1.4} = 2.5\ \Omega.$$

Thévenin equivalent:



Exercise 3-12 Find the Thévenin equivalent of the circuit to the left of terminals (a,b) in Fig. E3-12, and then determine the current I.

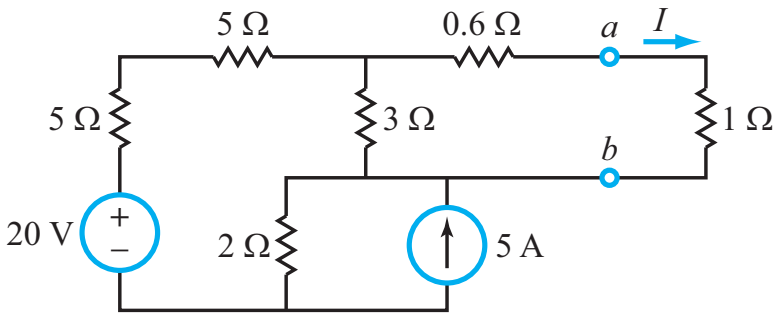
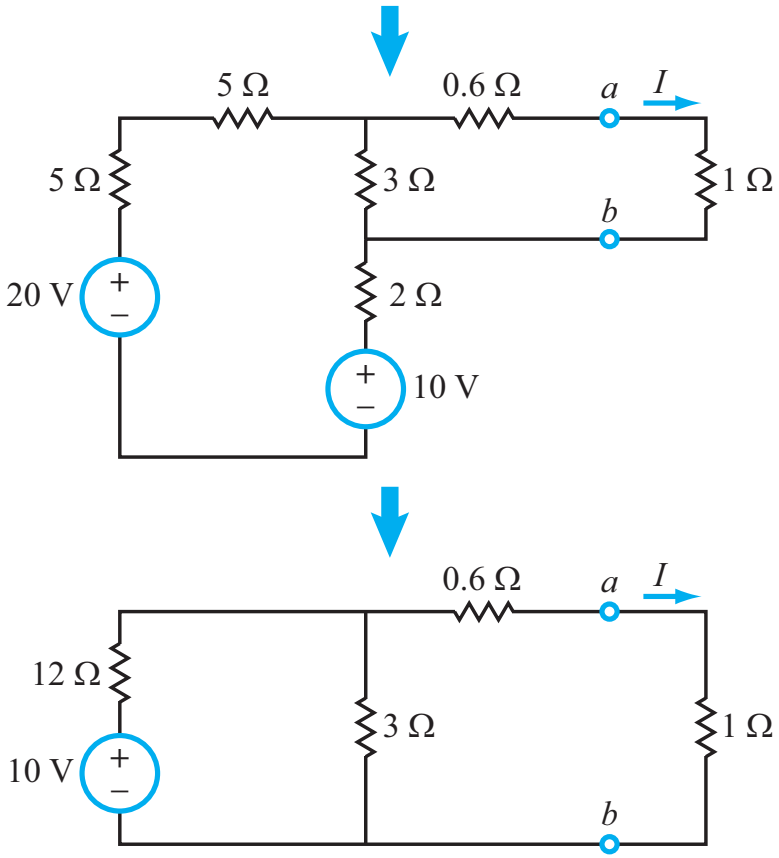


Figure E3-12

Solution: Since the circuit has no dependent sources, we will apply multiple steps of source transformation to simplify the circuit.



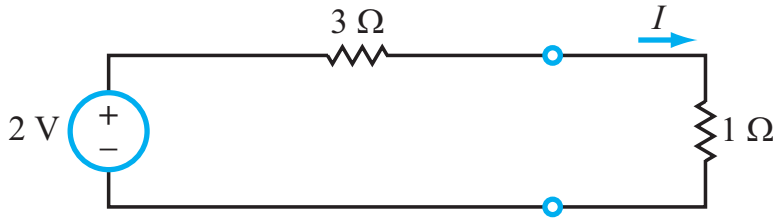
Across (a,b),

$$V_{\text{Th}} = V_{\text{oc}} = \frac{10 \times 3}{12+3} = 2 \text{ V}$$

$$R_{\text{Th}} = 3 \parallel 12 + 0.6$$

$$= \frac{3 \times 12}{3+12} + 0.6 = 3 \ \Omega$$

Hence,



$$I = \frac{2}{3+1} = 0.5 \text{ A}.$$

Exercise 3-13 Find the Norton equivalent at terminals (a, b) of the circuit in Fig. E3-13.

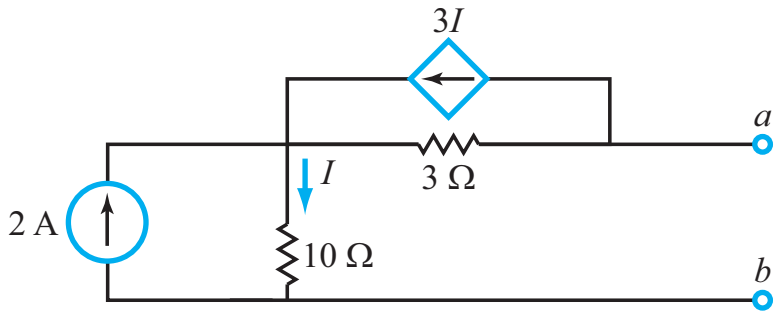
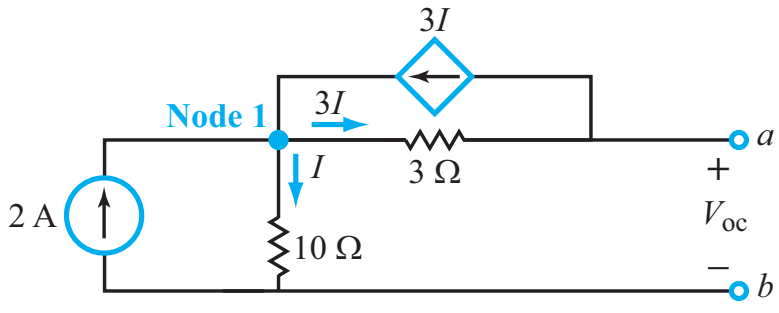


Figure E3-13

Solution: Thévenin voltage



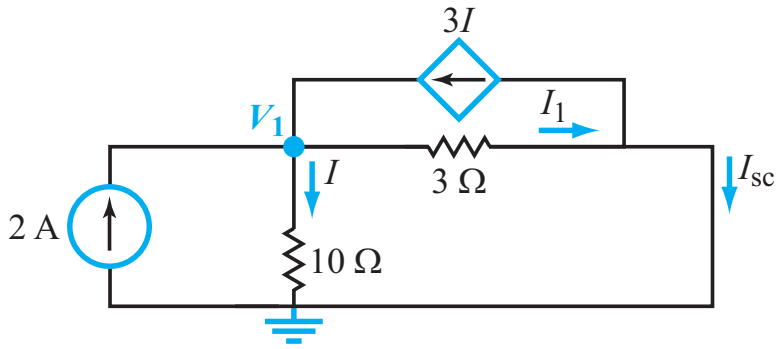
At node 1:

$$I = 2 \text{ A}.$$

Hence,

$$V_{Th} = V_{oc} = 10I - 3 \times 3I = I = 2 \text{ V}.$$

Next, we determine the short-circuit current:



At node V_1:

$$-2-3I+\frac{V_1}{10}+\frac{V_1}{3}=0.$$

Also,

$$I=\frac{V_1}{10}\,.$$

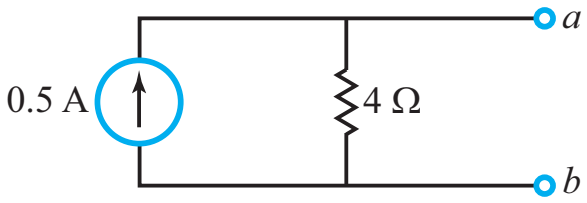
Hence,

$$-2-3I+I+\frac{10}{3}\,I=0,$$

which gives

$$\begin{aligned} I &= 1.5 \text{ A}, \\ I_1 &= 2+3I-I=2+2I=5 \text{ A}, \\ I_{sc} &= 5-3I=5-4.5=0.5 \text{ A}. \\ R_{Th} &= \frac{V_{Th}}{I_{sc}}=\frac{2}{0.5}=4\ \Omega. \end{aligned}$$

Norton circuit is:



Exercise 3-14 The bridge circuit of Fig. E3-14 is connected to a load R_L between terminals (a,b). Choose R_L such that maximum power is delivered to R_L. If $R = 3\ \Omega$, how much power is delivered to R_L?

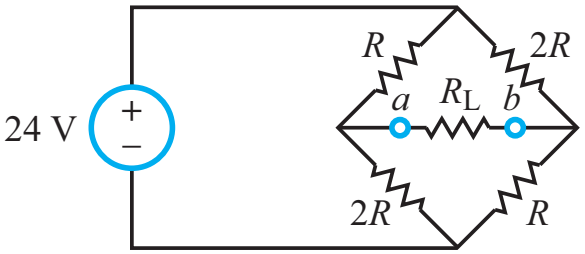
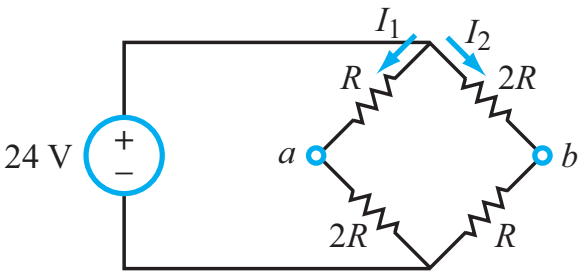


Figure E3-14

Solution: We need to remove R_L and then determine the Thévenin equivalent circuit at terminals (a,b).

Open-circuit voltage:

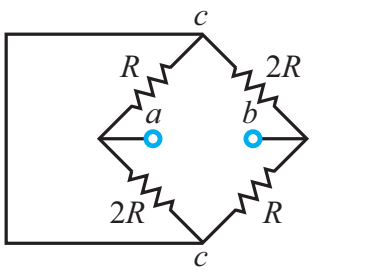


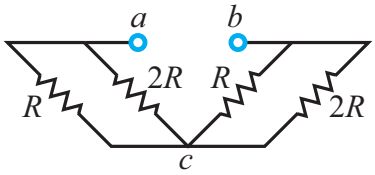
The two branches are balanced (contain same total resistance of $3R$). Hence, identical currents will flow, namely

$$I_1 = I_2 = \frac{24}{3R} = \frac{8}{R}\,.$$

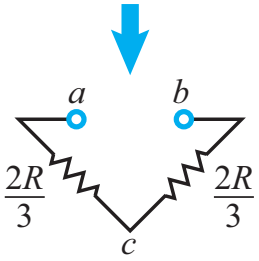
$$V_{oc} = V_a - V_b = 2RI_1 - RI_2 = RI_1 = R\,\frac{8}{R} = 8\ \text{V}.$$

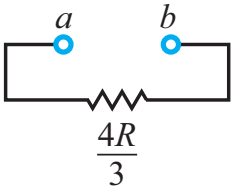
To find R_{Th}, we replace the source with a short circuit:





$$R \parallel 2R = \frac{R \times 2R}{R + 2R} = \frac{2}{3} R$$

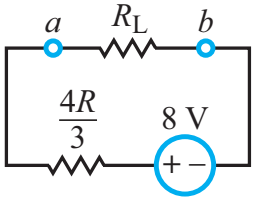




Hence,

$$R_{\text{Th}} = \frac{4R}{3},$$

and the Thévenin circuit is



For maximum power transfer with $R = 3\ \Omega$, R_L should be

$$R_L = \frac{4R}{3} = \frac{4 \times 3}{3} = 4\ \Omega,$$

and

$$P_{max} = \frac{v_s^2}{4R_L} = \frac{8^2}{4 \times 4} = 4\ \text{W}.$$

Exercise 3-15 Determine I_B, V_{out_1}, and V_{out_2} in the transistor circuit of Fig. E3-15, given that $V_{BE} = 0.7$ V and $\beta = 200$.

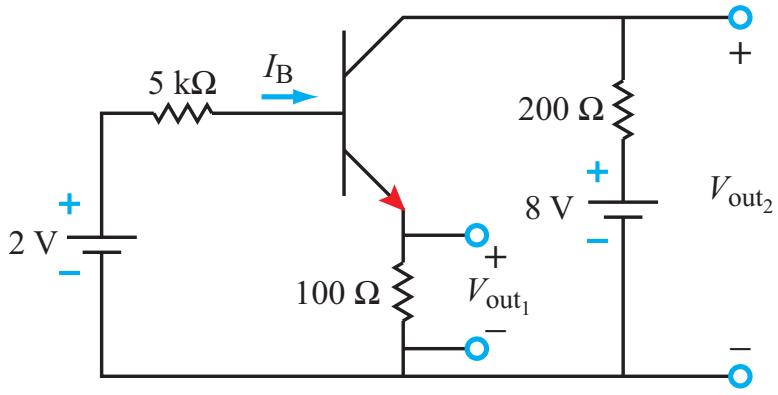
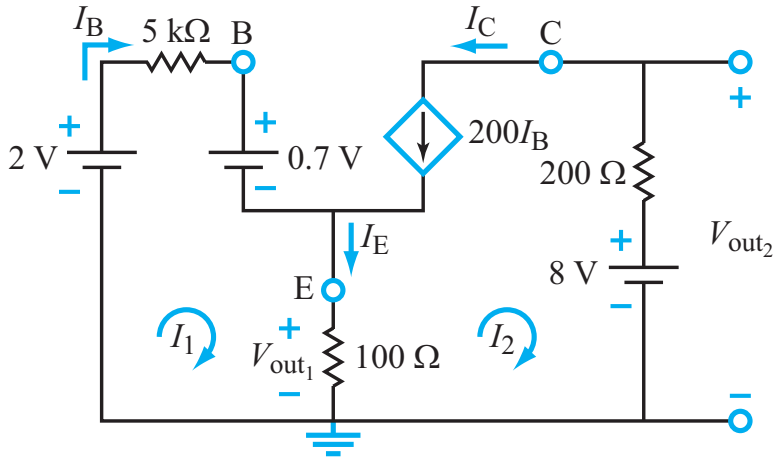


Figure E3.15

Solution: Using the equivalent-circuit model:



Loop 2

$$I_2 = -200I_B = -200I_1$$

Loop 1

$$-2 + I_1(5000) + 0.7 + 100(I_1 - I_2) = 0$$
$$-1.3 + 5100I_1 + 20000I_1 = 0$$

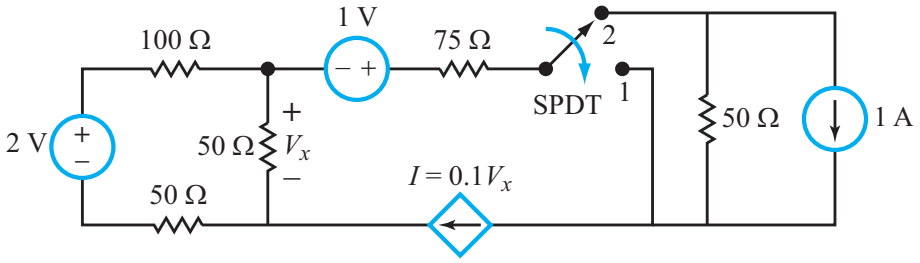
or

$$25100I_1 = 1.3,$$

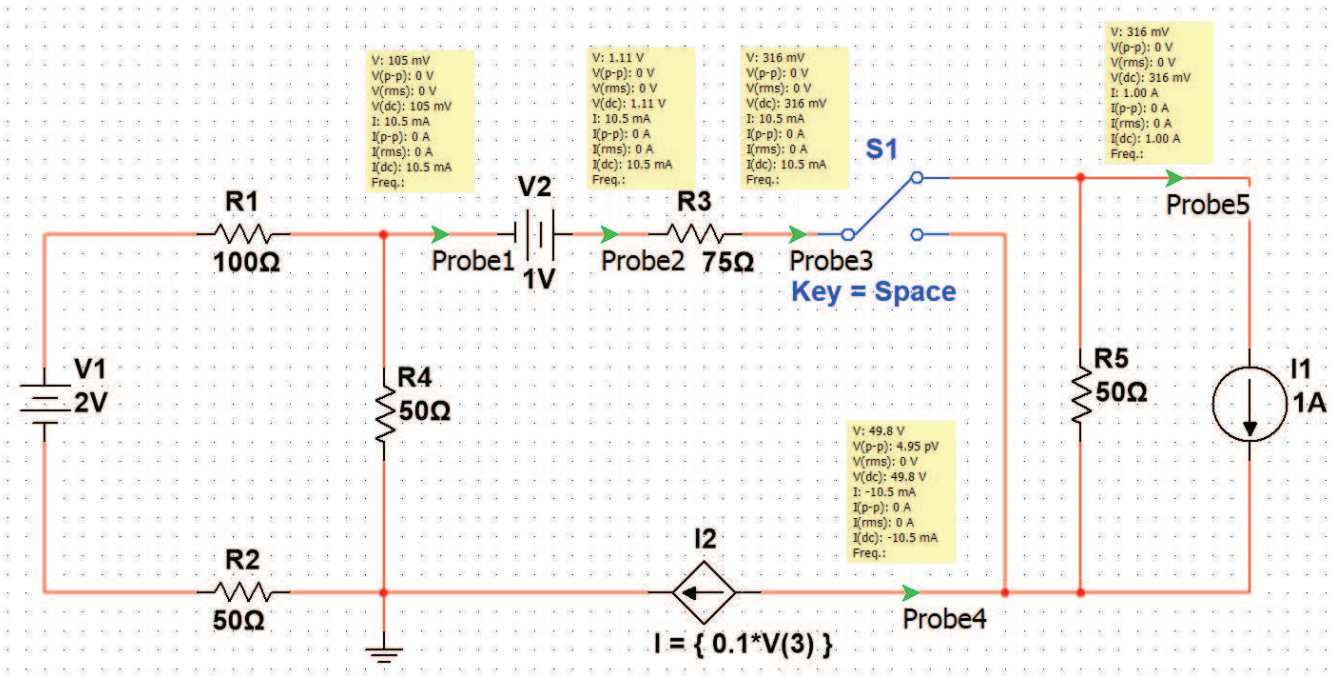
which gives

$$
\begin{aligned}
1.3 &= 25,100 I_1 \\
I_1 &= I_B = 0.00005179 \text{ A} = 51.79 \ \mu\text{A} \\
I_B &= 51.79 \ \mu\text{A} \\
I_C &= 200 I_B = 10.36 \text{ mA} \\
V_{\text{out}_2} &= 8 - 200 I_C = 8 - 2.07 = 5.93 \text{ V} \\
I_E &= I_B + I_C = 10.41 \text{ mA} \\
V_{\text{out}_1} &= 100 I_E = 1.041 \text{ V}.
\end{aligned}
$$

Exercise 3-16 Use Multisim to calculate the voltage at node 3 in Fig. 3-38(b) when the SPDT switch is connected to position 2.



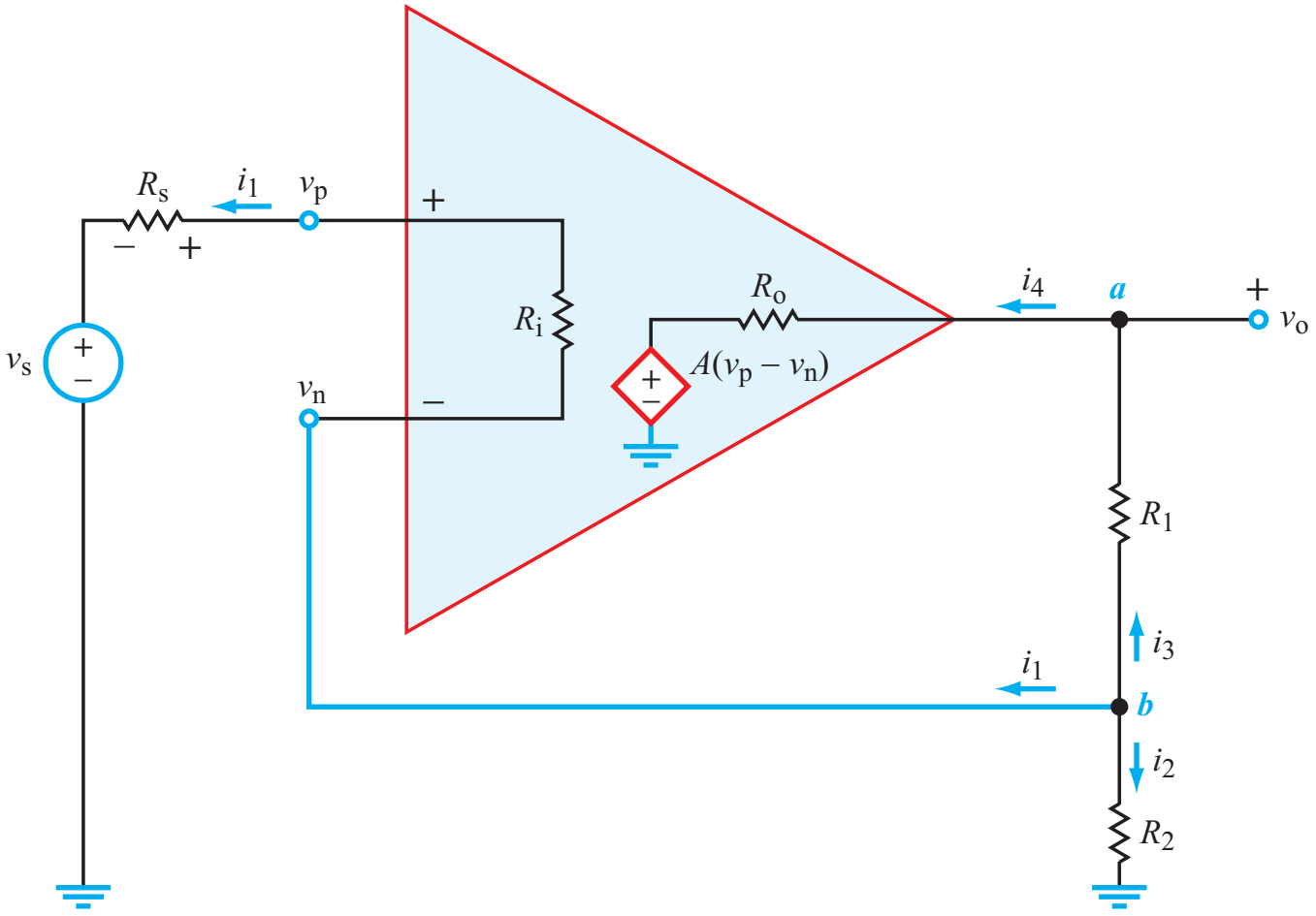
(a) Circuit with SPDT switch



(b) Multisim configuration

Solution: From Probe 1, we can see that V(3) = 13.0 mV.

Exercise 4-1 In the circuit of Example 4-1, shown in Fig. 4-5, insert a series resistance R_s between v_s and v_p, and then repeat the solution to obtain an expression for G. Evaluate G for $R_s = 10\ \Omega$, and use the same values listed in Example 4-1 for the other quantities. What impact does the insertion of R_s have on the magnitude of G?



Solution:

$$\text{At node } a: \quad \frac{v_n - v_o}{R_1} = \frac{v_o - A(v_p - v_n)}{R_o}.$$

$$\text{At node } b: \quad \frac{v_n - v_p}{R_i} + \frac{v_n}{R_2} + \frac{v_n - v_o}{R_1} = 0.$$

Additionally,

$$\begin{aligned} v_\mathrm{p} &= v_\mathrm{s} + i_1 R_\mathrm{s} \\ &= v_\mathrm{s} + \left(\frac{v_\mathrm{n} - v_\mathrm{p}}{R_\mathrm{i}} \right) R_\mathrm{s}. \end{aligned}$$

Solving the three equations simultaneously leads to

$$G = \frac{v_\mathrm{o}}{v_\mathrm{s}} = \frac{[A(R_\mathrm{i} + R_\mathrm{s})(R_1 + R_2) + R_2 R_\mathrm{o}]}{\begin{array}{r}[AR_2(R_\mathrm{i} + R_\mathrm{s}) + R_\mathrm{o}(R_2 + R_\mathrm{i} + R_\mathrm{s}) \\ + R_1 R_2 + (R_\mathrm{i} + R_\mathrm{s})(R_1 + R_2)]\end{array}}.$$

For $R_\mathrm{s} = 10\ \Omega$, $R_\mathrm{i} = 10^7\ \Omega$, $R_0 = 10\ \Omega$, $R_1 = 80\ \mathrm{k}\Omega$, $R_2 = 20\ \mathrm{k}\Omega$, and $A = 10^6$,

$$G = 4.999977 \simeq 5.0.$$

Exercise 4-2 To evaluate the tradeoff between the circuit gain G and the linear dynamic range of υ_s, apply Eq. (4.8) to find the magnitude of G, and then determine the corresponding dynamic range of υ_s, for each of the following values of R_2: 0 (no feedback), 800 Ω, 8.8 kΩ, 40 kΩ, 80 kΩ, 1 MΩ. Except for R_2, all other quantities remain unchanged.

Solution:

$$G = \frac{AR_i(R_1 + R_2) + R_2R_o}{AR_2R_i + R_o(R_2 + R_i) + R_1R_2 + R_i(R_1 + R_2)}$$

$$\upsilon_o = G\upsilon_s$$

$$|\upsilon_o|_{max} = \upsilon_{cc} = 10 \text{ V}.$$

Hence,

$$|\upsilon_s|_{max} = \frac{10}{G}.$$

For $A = 10^6$, $R_i = 10^7$ Ω, $R_1 = 80$ kΩ, $R_o = 10$ Ω, we obtain the following table:

R_2	G	υ_s Range
0	10^6	-10 μV to $+10$ μV
800 Ω	101	-99 mV to $+99$ mV
8.8 kΩ	10.1	-0.99 V to $+0.99$ V
40 kΩ	3	-3.3 V to $+3.3$ V
80 kΩ	2	-5 V to $+5$ V
1 MΩ	1.08	-9.26 V to $+9.26$ V

Exercise 4-3 Consider the noninverting amplifier circuit of Fig. 4-9(a), under the conditions of the ideal op-amp model. Assume $V_{cc} = 10$ V. Determine the value of G and the corresponding dynamic range of υ_s for each of the following values of R_1/R_2: 0, 1, 9, 99, 10^3, 10^6.

Solution:

$$G = \frac{\upsilon_o}{\upsilon_s} = \frac{R_1 + R_2}{R_2} ,$$

$$|\upsilon_o|_{max} = V_{cc} = 10 \text{ V} ,$$

$$|\upsilon_s|_{max} = \frac{10}{G} .$$

Using these expressions leads to the following table:

R_1/R_2	G	υ_s Range
0	1	-10 V to $+10$ V
1	2	-5 V to $+5$ V
9	10	-1 V to $+1$ V
99	100	-0.1 V to $+0.1$ V
1000	≈ 1000	-10 mV to $+10$ mV (Approx.)
10^6	$\approx 10^6$	-10 μV to $+10$ μV (Approx.)

Exercise 4-4 The input to an inverting-amplifier circuit consists of $v_s = 0.2$ V and $R_s = 10\ \Omega$. If $V_{cc} = 12$ V, what is the maximum value that R_f can assume before saturating the op amp?

Solution:

$$G = -\frac{R_f}{R_s},$$

$$|v_o|_{max} = V_{cc} = 12 \text{ V}.$$

At saturation

$$|v_s| = \frac{|v_o|_{max}}{G}.$$

Hence, maximum allowed value of $|G|$ is

$$|G| = \frac{|v_o|_{max}}{v_s} = \frac{12}{0.2} = 60,$$

which corresponds to

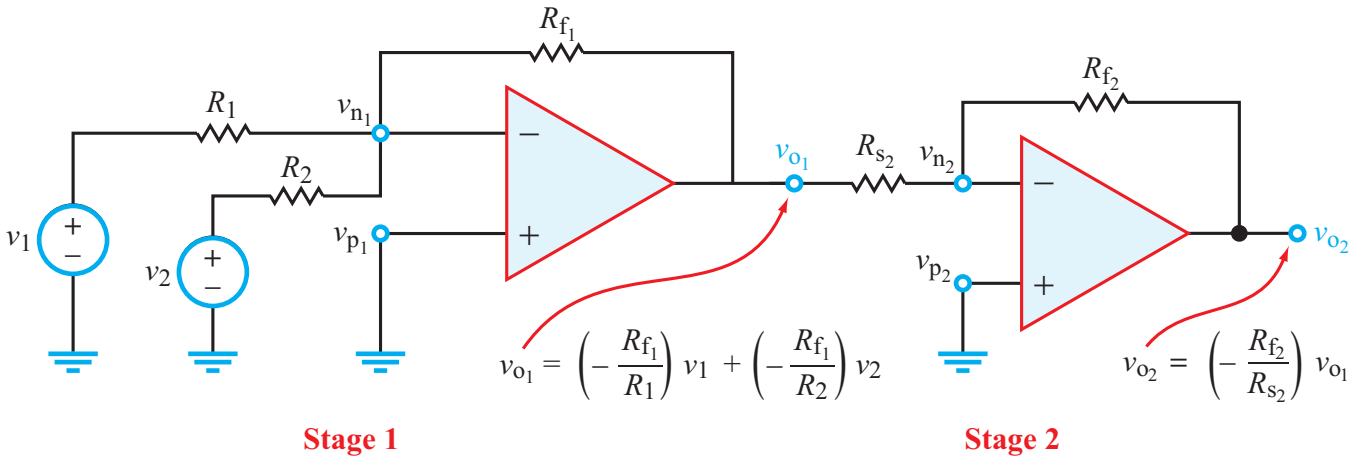
$$R_f = |G|R_s = 60 \times 10 = 600\ \Omega.$$

Exercise 4-5 The circuit shown in Fig. 4-13(a) is to be used to perform the operation

$$v_o = 3v_1 + 6v_2.$$

If $R_1 = 1.2\ \text{k}\Omega$, $R_{s_2} = 2\ \text{k}\Omega$, and $R_{f_2} = 4\ \text{k}\Omega$, select values for R_2 and R_{f_1} so as to realize the desired result.

Solution:



(a) Two-stage circuit

Given the output-input relations for the two stages, it follows that

$$v_{o_2} = \left(-\frac{R_{f_2}}{R_{s_2}}\right) v_{o_1} = \left(-\frac{R_{f_2}}{R_{s_2}}\right) \left[\left(-\frac{R_{f_1}}{R_1}\right) v_1 + \left(-\frac{R_{f_1}}{R_2}\right) v_2\right],$$

$$= \left(\frac{R_{f_1} R_{f_2}}{R_1 R_{s_2}}\right) v_1 + \left(\frac{R_{f_1} R_{f_2}}{R_2 R_{s_2}}\right) v_2.$$

We are given that $R_1 = 1.2\ \text{k}\Omega$, $R_{s_2} = 2\ \text{k}\Omega$, and $R_{f_2} = 4\ \text{k}\Omega$. Additionally, to match the required operation, we need to have

$$\frac{R_{f_1} R_{f_2}}{R_1 R_{s_2}} = 3,$$

$$\frac{R_{f_1} R_{f_2}}{R_2 R_{s_2}} = 6.$$

The ratio of these two conditions gives

$$\frac{R_2}{R_1} = \frac{1}{2}, \qquad \text{or} \qquad R_2 = \frac{R_1}{2} = \frac{1200}{2} = 600\ \Omega.$$

Finally,

$$\frac{4 \times 10^3 R_{f_1}}{1.2 \times 10^3 \times 2 \times 10^3} = 3$$

leads to

$$R_{f_1} = 1.8\ \text{k}\Omega.$$

Exercise 4-6 The difference-amplifier circuit of Fig. 4-15 is used to realize the operation

$$v_o = (6v_2 - 2)\ \text{V}.$$

Given that $R_3 = 5\ \text{k}\Omega$, $R_4 = 6\ \text{k}\Omega$, and $R_2 = 20\ \text{k}\Omega$, specify values for v_1 and R_1.

Solution:

$$v_o = \left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{R_1 + R_2}{R_1}\right)v_2 - \frac{R_2}{R_1}v_1\ .$$

To satisfy the required operation, it is necessary that

$$\left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{R_1 + R_2}{R_1}\right) = 6.$$

Given that $R_3 = 5\ \text{k}\Omega$, $R_4 = 6\ \text{k}\Omega$, and $R_2 = 20\ \text{k}\Omega$, it follows that

$$R_1 = 2\ \text{k}\Omega.$$

To satisfy the second term of the operation, we need to have

$$\left(\frac{R_2}{R_1}\right)v_1 = 2,$$

or

$$v_1 = \frac{2R_1}{R_2} = \frac{2 \times 2 \times 10^3}{20 \times 10^3} = 0.2\ \text{V}.$$

Exercise 4-7 Express v_o in terms of v_1, v_2 and v_3 for the circuit in Fig. E4-7.

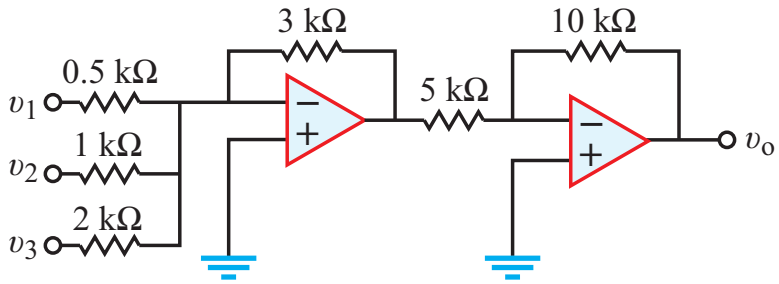


Figure E4-7

Solution: Starting from the output of the second stage and moving backwards towards the inputs,

$$v_o = \left(-\frac{10\times10^3}{5\times10^3}\right)\left[\left(-\frac{3\times10^3}{0.5\times10^3}\right)v_1 + \left(\frac{-3\times10^3}{10^3}\right)v_2 + \left(\frac{-3\times10^3}{2\times10^3}\right)v_3\right]$$
$$= 12v_1 + 6v_2 + 3v_3.$$

Exercise 4-8 To monitor brain activity, an instrumentation-amplifier sensor uses a pair of needle-like probes, inserted at different locations in the brain, to measure the voltage difference between them. If the circuit is of the type shown in Fig. 4-22, with $R_1 = R_3 = R_4 = R_5 = R = 50\ \text{k}\Omega$ and $V_{cc} = 12$ V, and the maximum magnitude of the voltage difference that the brain is likely to exhibit is 3 mV, what should R_2 be to maximize the sensitivity of the brain sensor?

Solution:

$$v_o = \left(1 + \frac{2R}{R_2}\right)(v_2 - v_1).$$

We are given that $v_2 - v_1 = 3$ mV and to avoid saturation $|v_o|_{max}$ should not exceed V_{cc}. Hence,

$$1 + \frac{2R}{R_2} = \frac{|v_o|_{max}}{|v_2 - v_1|} = \frac{12}{3 \times 10^{-3}} = 4000,$$

and

$$R_2 + \frac{2R}{4000 - 1} \approx \frac{2 \times 50 \times 10^3}{4 \times 10^3} = 25\ \Omega.$$

Exercise 4-9 A 3-bit DAC uses an R-$2R$ ladder design with $R = 3\ \text{k}\Omega$ and $R_\text{f} = 24\ \text{k}\Omega$. If $V_\text{cc} = 10$ V, write an expression for V_out and evaluate it for $[V_1V_2V_3] = [111]$.

Solution:

$$V_\text{out} = -\left(\frac{R_\text{f}}{R_\text{Th}}\right) V_\text{Th},$$

with

$$R_\text{Th} = R,$$

and

$$V_\text{Th} = \frac{V_1}{2} + \frac{V_2}{4} + \frac{V_3}{8}\ .$$

With $R_\text{f} = 24\ \text{k}\Omega$ and $R = 3\ \text{k}\Omega$,

$$V_\text{out} = -(4V_1 + 2V_2 + V_3).$$

For $[V_1V_2V_3] = [111]$, $V_\text{out} = -7$ V.

Exercise 4-10 In the circuit of Example 4-9, what value of R_D will give the highest possible ac gain, while keeping $v_{out}(t)$ always positive?

Solution:

$$\begin{aligned} V_{out} &= V_{DD} - gR_D\, v_s(t) \\ &= V_{DD} - gR_D(500 + 40\cos 300t) \times 10^{-6} \\ &= (V_{DD} - 500gR_D \times 10^{-6}) - 40gR_D \times 10^{-6}\cos 300t. \end{aligned}$$

Maximizing the ac gain without allowing v_{out} to go negative is accomplished by selecting R_D such that

$$\frac{40gR_D \times 10^{-6}}{V_{DD} - 500gR_D \times 10^{-6}} = 1.$$

Given that $V_D = 10$ V and $g = 10$ A/V, it follows that

$$R_D = 1.85\ \text{k}\Omega.$$

Exercise 4-11 Repeat Example 4-10, but require that v_{out} be at least 99.9% of v_s. What should R_L be: (a) without the buffer, and (b) with the buffer?

Solution:

Without Buffer

$$\frac{R_L}{R_s} = 999, \qquad \text{or } R_L = 999 \times 100 = 99.9 \text{ k}\Omega.$$

With Buffer

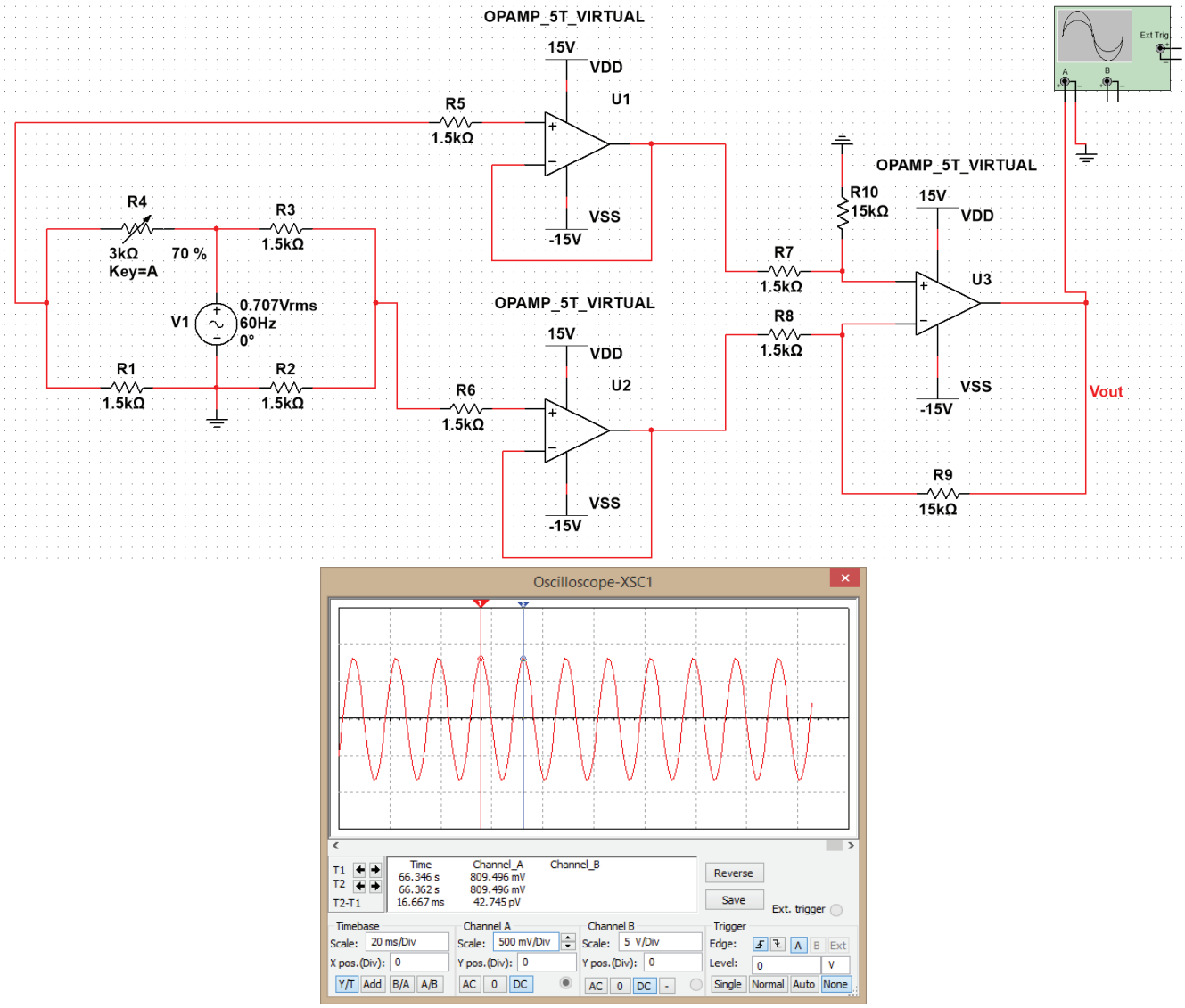
$$\frac{v_o}{v_s} = 0.999 = \frac{9R_L}{1 + gR_L}.$$

With $g = 10$ A/V,

$$R_L = 99.9 \ \Omega.$$

Exercise 4-12 Why are the voltage followers necessary in the circuit of Fig. 4-35? Remove them from the Multisim circuit and connect the resistive bridge directly to the two inputs of the differential amplifier. How does the output vary with the potentiometer setting?

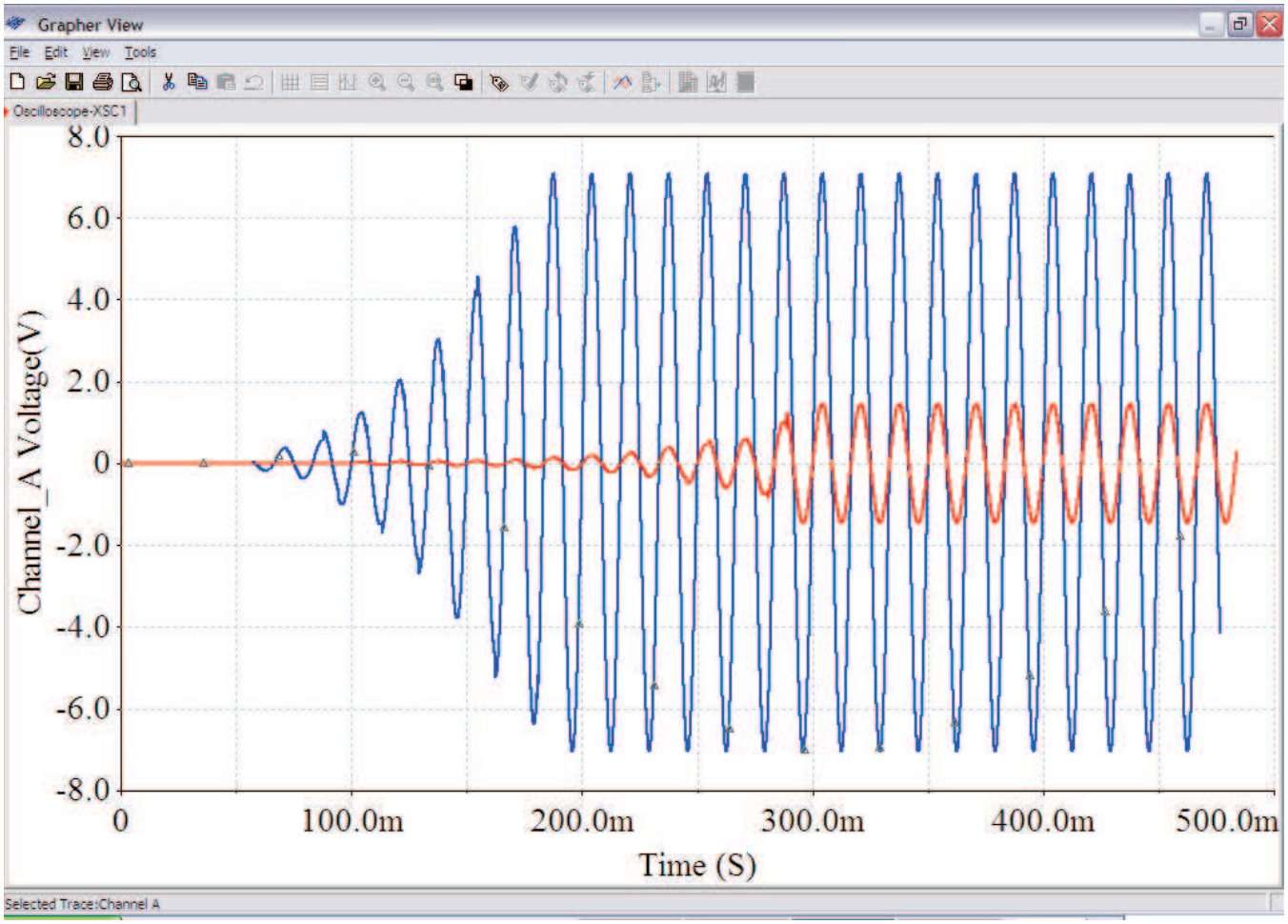
Solution:



The voltage followers are necessary because they have a high input impedance; this high impedance prevents any currents from running between the bridge and the high gain amplifier (particularly resistor R8 in Fig. 4-33), effectively isolating the two components. Consider our analysis of the Wheatstone bridge in Section 2-6; applying the same analysis to Fig. 4-33 assumes that all of the current through R3 flows through R5 and all of the current through R1 flows through R2. This assumption is only true if the input impedance seen across

node 3 and node 4 is much higher than the resistances of the bridge (otherwise, non-negligible current will flow through the op-amps, lowering the value of the voltage between node 3 and node 4).

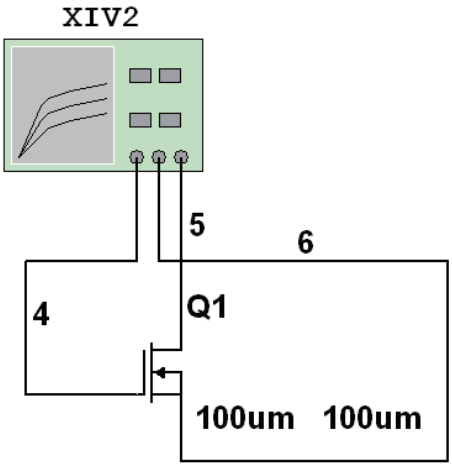
Removing the two voltage followers will result in a lower voltage across nodes 3 and 4, and thus, a lower output voltage. This is shown in the figure below.



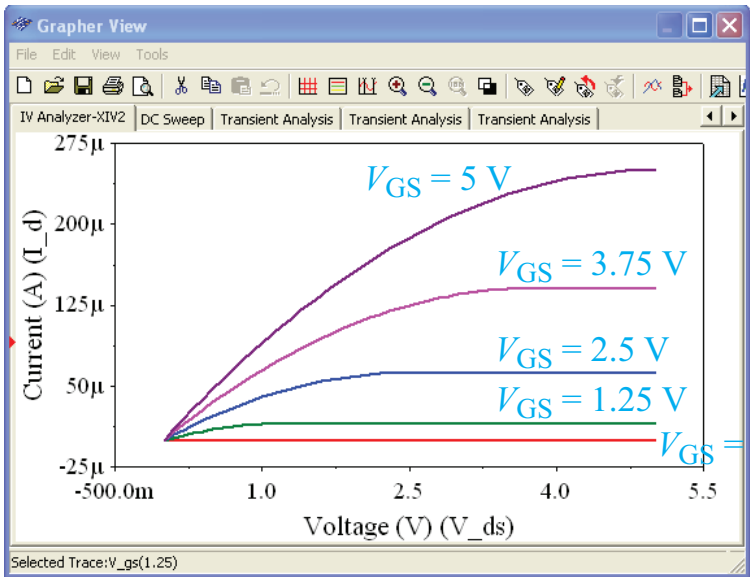
The best way to see this is to remove the two voltage followers from the circuit and raise the resistances of all four Wheatstone bridge resistors to, say, 100 kΩ. Re-run the Interactive Simulation. Notice the difference in the voltage output *with* (blue line) and *without* (red line) the voltage followers; when plotting this figure, R3 was varied from an initial value of 100% (100 kΩ) to 0% (0% kΩ). In the author's computer, TMAX and the Initial Time Step were set to 1e-5 s under Simulate ⟶ Interactive Simulation Settings.

Exercise 4-13

The ***I-V Analyzer*** is another useful Multisim instrument for analyzing circuit performance. To demonstrate its utility, let us use it to generate characteristic curves for an NMOS transistor, similar to those in Fig. 4-27(b). Figure E4-13(a) shows an NMOS connected to an I-V Analyzer. The instrument sweeps through a range of gate (G) voltages and generates a current-versus-voltage (I-V) plot between the drain (D) and source (S) for each gate voltage. Show that the display of the I-V analyzer is the same as that shown in Fig. E4-13(b).



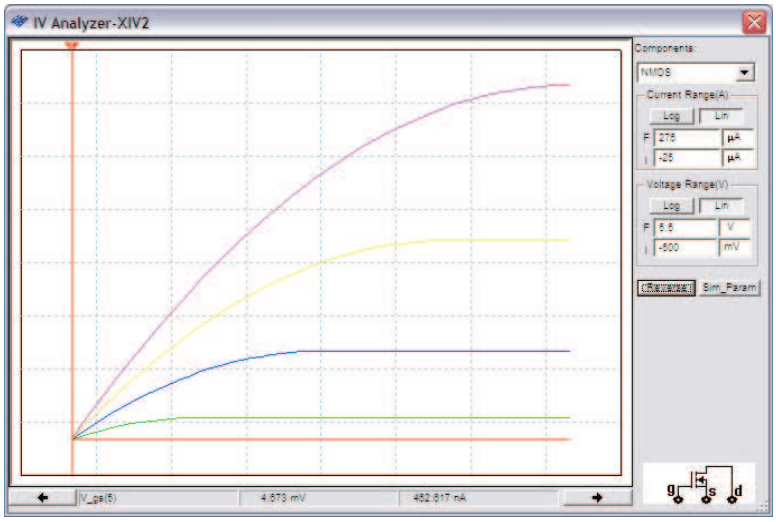
(a)



(b)

Fig. E4-13: (a) Circuit schematic and (b) I-V analyzer traces for I_{DS} versus V_{DS} at selected values of V_{GS}.

Solution: Draw your circuit as in Fig. E4-13(a). Double click on the IV Analyzer; it should look like the figure below.



Select NMOS under the Components drop-down menu; press the Lin (linear plot) button for both the Current Range and Voltage Range. Set current and voltage values as shown in the figure. You can generate Figure E4-13(b) by opening the Grapher windows and formatting accordingly.

Exercise 5-1 Express the waveforms shown in Fig. E5-1 in terms of unit step functions.

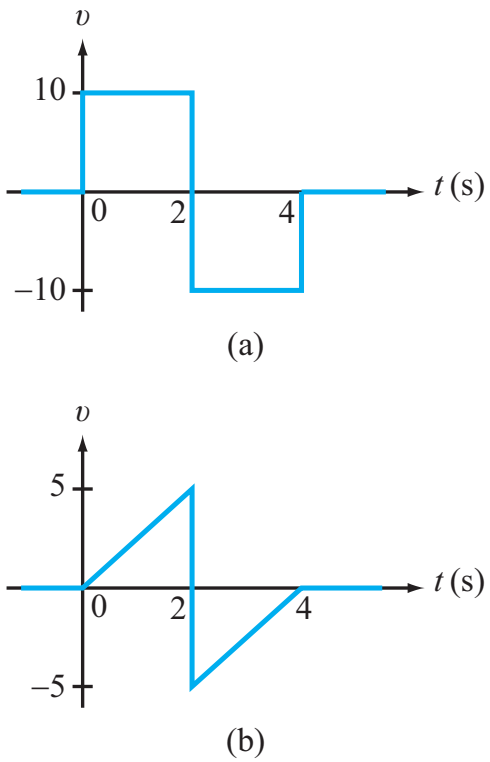
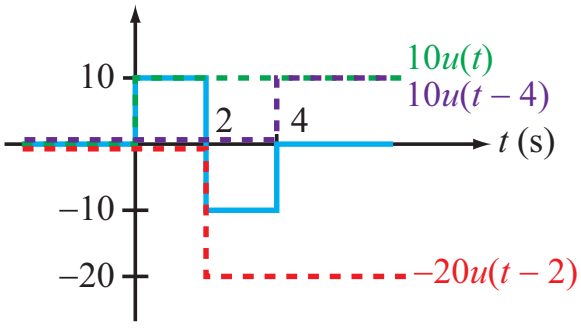


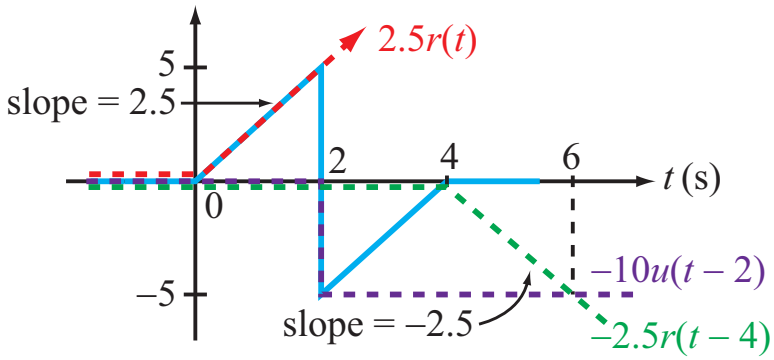
Figure E5-1

Solution:
(a)



$v(t) = 10u(t) - 20u(t-2) + 10u(t-4).$

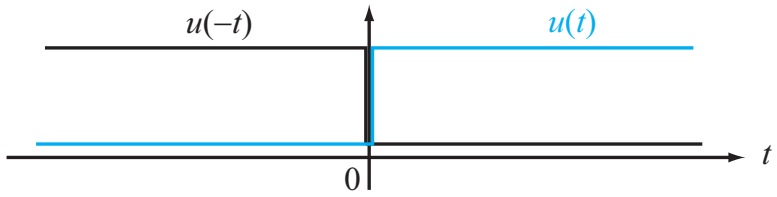
(b)



$v(t) = 2.5r(t) - 10u(t-2) - 2.5r(t-4).$

Exercise 5-2 How is $u(t)$ related to $u(-t)$?

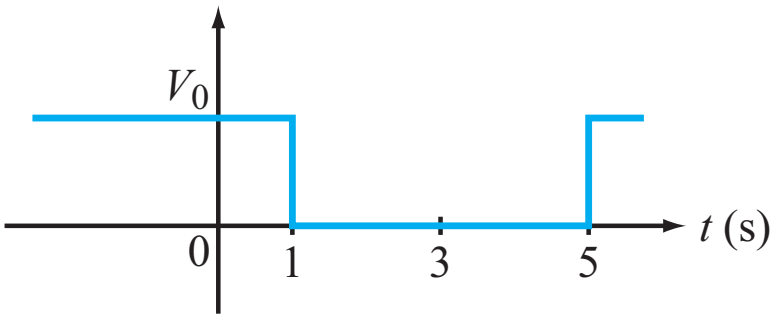
Solution:



$u(-t)$ is the mirror image of $u(t)$ with respect to the vertical axis.

Exercise 5-3 Consider the SPDT switch in Fig. 5-6(a). Assume that it started out at position 2, was moved to position 1 at $t = 1$ s, and then moved back to position 2 at $t = 5$ s. This is the reverse of the sequence shown in Fig. 5-6(a). Express $\upsilon(t)$ in terms of (a) units step functions and (b) the rectangle function.

Solution:



(a) $\upsilon(t) = V_0[u(1-t) + u(t-5)]$
(b) $\upsilon(t) = V_0 - V_0 \,\text{rect}\left(\frac{t-3}{4}\right)$ [rectangle center at 3 s and length = 4 s].

Exercise 5-4 The radioactive decay equation for a certain material is given by $n(t) = n_0 e^{-t/\tau}$, where n_0 is the initial count at $t = 0$. If $\tau = 2 \times 10^8$ s, how long is its half-life? [Half-life $t_{1/2}$ is the time it takes a material to decay to 50% of its initial value.]

Solution: Given

$$n(t) = n_0 e^{-t/(2\times 10^8)},$$

The time t at which $n(t) = n_0/2$ is obtained by solving for t in

$$\frac{n_0}{2} = n_0 e^{-t/(2\times 10^8)},$$

or

$$\ln \frac{1}{2} = \frac{-t}{2 \times 10^8},$$

which gives $t = -2 \times 10^8 \ln 2 = 1.386 \times 10^8$ s = 4 years, 144 days, 12 hours, 10 minutes, and 36 s.

Exercise 5-5 If the current $i(t)$ through a resistor R decays exponentially with a time constant τ, what is the value of the power dissipated in the resistor at $t = \tau$, compared with its value at $t = 0$?

Solution:

$$p(t) = i^2 R = I_0^2 R(e^{-t/\tau})^2 = I_0^2 R e^{-2t/\tau} ,$$

$$\left(\frac{p(\tau)}{p(0)}\right) = e^{-2} = 0.135, \qquad \text{or } 13.5\%.$$

Exercise 5-6 It is desired to build a parallel-plate capacitor capable of storing 1 mJ of energy when the voltage across it is 1 V. If the capacitor plates are 2 cm $\times$ 2 cm each, and its insulating material is Teflon, what shold the separation d be? Is such a capacitor practical?

Solution:

$$w = \frac{1}{2} C v^2,$$

$$10^{-3} = \frac{1}{2} C 1^2, \quad \Longrightarrow \quad C = 2 \times 10^{-3} \text{ F},$$

$$C = \frac{\varepsilon A}{d}, \qquad \text{and } \varepsilon = 2.1\varepsilon_0 \text{ for Teflon.}$$

Hence,

$$2 \times 10^{-3} = \frac{2.5 \times 8.85 \times 10^{-12} \times (0.02)^2}{d},$$

which yields

$$d = 3.72 \times 10^{-12} \text{ m}.$$

Certainly, not practical.

Exercise 5-7 Instead of specifying A and calculating the spacing d needed to meet the 1-mJ requirement in Exercise 5-6, suppose we specify d as 1 μm and then calculate A. How large would A have to be?

Solution:

$$C = 0.002 = \frac{2.1 \times 8.85 \times 10^{-12}\ \text{A}}{10^{-6}},$$

which gives

$$A = 10.4\ \text{m} \times 10.4\ \text{m}.$$

Equally impractical size.

Exercise 5-8 Determine the current i in the circuit of Fig. E5-8, under dc conditions.

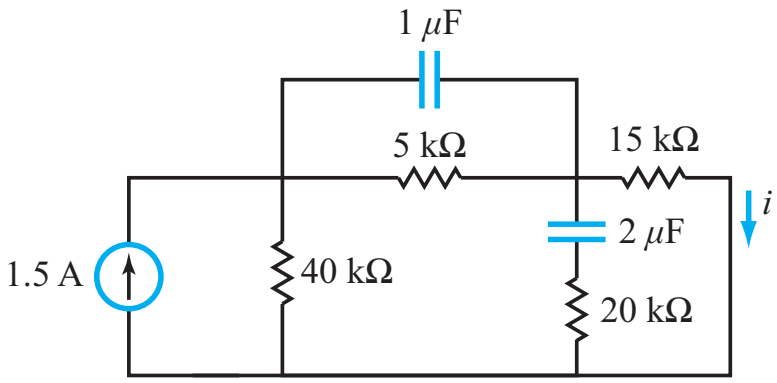
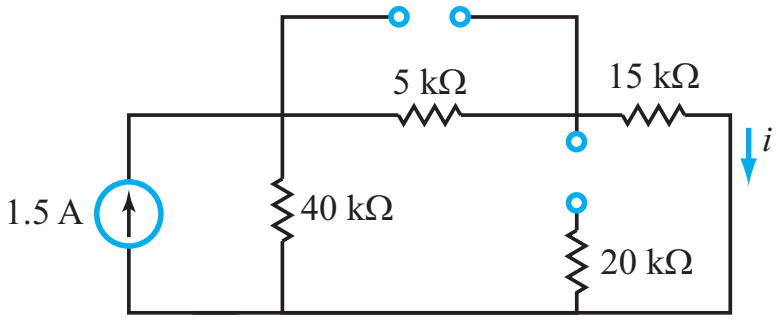


Figure E5-8

Solution: Under dc conditions, capacitors act like open circuits. Hence, the circuit becomes:



Voltage division gives

$$i = 1.5 \times \frac{40\text{k}}{40\text{k} + 15\text{k} + 5\text{k}} = 1 \text{ A}.$$

Exercise 5-9 Determine C_{eq} and $V_{eq}(0)$ at terminals (a,b) for the circuit in Fig. E5-9, given that $C_1 = 6\ \mu$F, $C_2 = 4\ \mu$F and $C_3 = 8\ \mu$F, and the initial voltages on the three capacitors are $v_1(0) = 5$ V and $v_2(0) = v_3(0) = 10$ V.

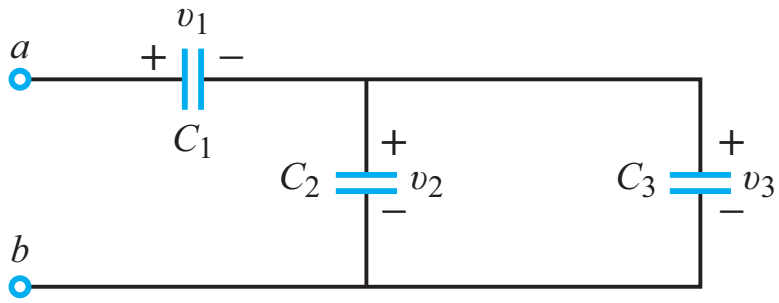


Figure E5-9

Solution:

$$C_{eq} = \frac{C_1(C_2 \parallel C_3)}{C_1 + C_2 + C_3}$$

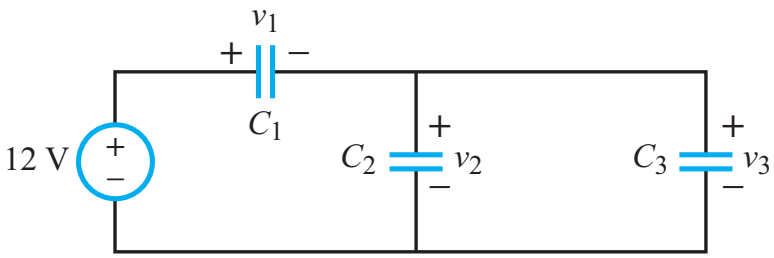
$$= \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$$

$$= \frac{6 \times 10^{-6}(4 \times 10^{-6} + 8 \times 10^{-6})}{(6+4+8) \times 10^{-6}} = 4\ \mu\text{F},$$

$$V_{eq}(0) = v_1(0) + v_2(0) = 5 + 10 = 15\ \text{V}.$$

Exercise 5-10 Suppose the circuit of Fig. E5-9 is connected to a dc voltage source $V_0 = 12$ V. Assuming that the capacitors had no charge before they were connected to the voltage source, determine v_1 and v_2, given that $C_1 = 6\ \mu$F, $C_2 = 4\ \mu$F, and $C_3 = 8\ \mu$F.

Solution:



According to Eq. (5.46),

$$C_1 v_1 = (C_2 \parallel C_3) v_2,$$

or

$$v_2 = \frac{C_1 v_1}{C_2 + C_3} = \frac{6 \times 10^{-6}}{4 \times 10^{-6} + 8 \times 10^{-6}} v_1 = \frac{v_1}{2}.$$

But

$$v_1 + v_2 = 12 \text{ V}.$$

Hence,

$$v_1 = 8 \text{ V} \qquad \text{and} \qquad v_2 = 4 \text{ V}.$$

Exercise 5-11 Calculate the inductance of a 20-turn air-core solenoid if its length is 4 cm and the radius of its circular cross section is 0.5 cm.

Solution:

$$L = \frac{\mu N^2 S}{\ell} = \frac{4\pi \times 10^{-7} \times 20^2 \times \pi(0.005)^2}{0.04}$$
$$= 0.987\ \mu\text{H}.$$

Exercise 5-12 Determine currents i_1 and i_2 in the circuit of Fig. E5-12, under dc conditions.

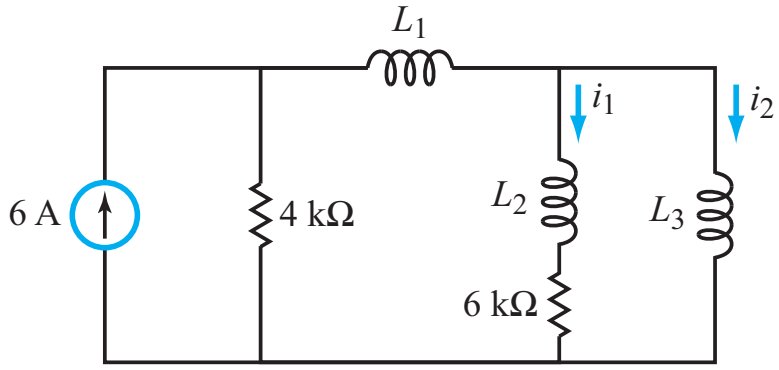
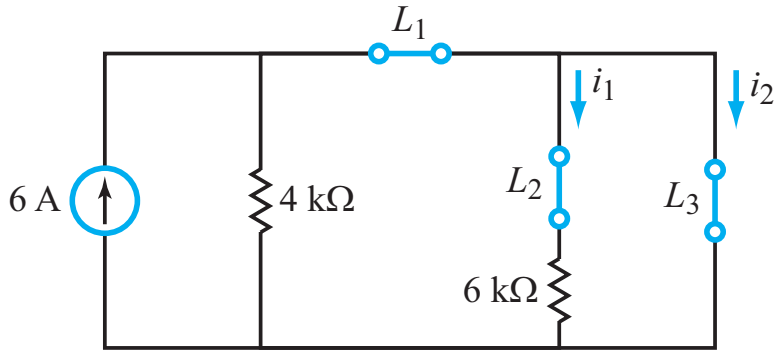


Figure E5-12

Solution: Under dc conditions, inductors act like short circuits.



The 6-A current will flow entirely through the short circuit representing L_3. Hence,

$$i_1 = 0, \qquad i_2 = 6 \text{ A}.$$

Exercise 5-13 Determine L_{eq} at terminals (a,b) in the circuit of Fig. E5-13.

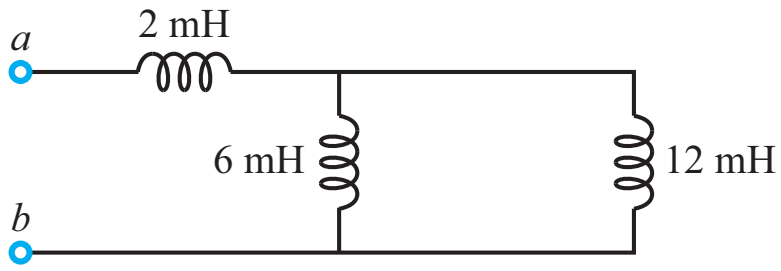


Figure E5-13

Solution:

$$\begin{aligned} L_{eq} &= 2\text{ mH} + (6\text{ mH} \parallel 12\text{ mH}) \\ &= \left(2 + \frac{6 \times 12}{6+12}\right)\text{ mH} \\ &= 6\text{ mH}. \end{aligned}$$

Exercise 5-14 If in the circuit of Fig. E5-14, $v(0^-) = 24$ V, determine $v(t)$ for $t \geq 0$.

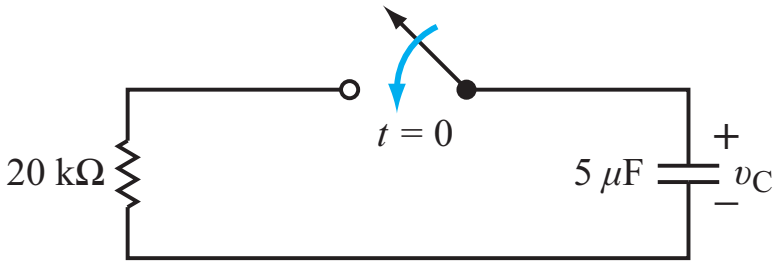


Figure E5-14

Solution:

$$\begin{aligned} v(t) &= v(0)\, e^{-t/\tau} \\ &= v(0)\, e^{-t/RC} \\ &= 24e^{-10t} \text{ V}, \qquad \text{for } t \geq 0. \end{aligned}$$

Exercise 5-15 Determine $v_1(t)$ and $v_2(t)$ for $t \geq 0$, given that in the circuit of Fig. E5-15 $C_1 = 6\ \mu$F, $C_2 = 3\ \mu$F, $R = 100\ \text{k}\Omega$, and neither capacitor had any charge prior to $t = 0$.

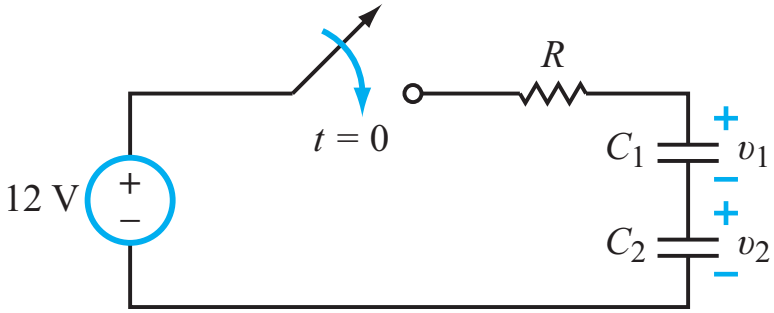


Figure E5-15

Solution:

$$v_1(0) = v_2(0) = 0 \qquad \text{[given]}$$
$$v_1(\infty) + v_2(\infty) = 12\ \text{V} \qquad \text{[At } t = \infty\text{, capacitors act like open circuits]}$$
$$C_1 v_1(\infty) = C_2 v_2(\infty), \qquad \text{[Eq. (5.46)].}$$

Hence,

$$C_1 v_1(\infty) = C_2[12 - v_1(\infty)],$$

which leads to

$$v_1(\infty) = 12\,\frac{C_2}{C_1 + C_2} = 4\ \text{V},$$
$$v_2(\infty) = 12 - 4 = 8\ \text{V}.$$

Also,

$$\tau = RC_{\text{eq}} = R\,\frac{C_1 C_2}{C_1 + C_2} = 0.2\ \text{s}.$$

Hence,

$$v_1(t) = v_1(\infty) + [v_1(0) - v_1(\infty)]e^{-t/\tau}$$
$$= 4(1 - e^{-5t})\ \text{V}, \qquad \text{for } t \geq 0,$$
$$v_2(t) = v_2(\infty) + [v_2(0) - v_2(\infty)]e^{-t/\tau}$$
$$= 8(1 - e^{-5t})\ \text{V}, \qquad \text{for } t \geq 0.$$

Exercise 5-16 Determine $i_1(t)$ and $i_2(t)$ for $t \geq 0$, given that in the circuit of Fig. E5-16 $L_1 = 6$ mH, $L_2 = 12$ mH, and $R = 2\ \Omega$. Assume $i_1(0^-) = i_2(0^-) = 0$.

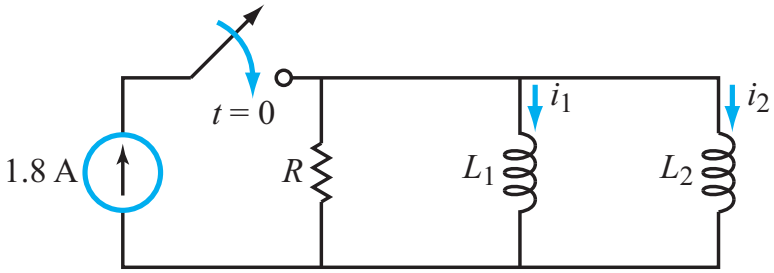


Figure E5-16

Solution:

$$
\begin{aligned}
i_1(t) &= \frac{1}{L_1}\int_0^t v(t)\,dt \\
&= \frac{1.8R}{L_1}\int_0^t e^{-500t}\,dt \\
&= \frac{1.8R}{L_1}\left[\frac{e^{-500t}}{-500}\right]_0^t \\
&= \frac{1.8\times 2}{500L_1}\left(1-e^{-500t}\right) \\
&= 1.2(1-e^{-500t})\,u(t)\ \text{A}, \\
i_2(t) &= \frac{1}{L_2}\int_0^t v(t)\,dt \\
&= \frac{1}{12\times 10^{-3}}\int_0^t v(t)\,dt \\
&= 0.6(1-e^{-500t})\,u(t)\ \text{A}.
\end{aligned}
$$

Exercise 5-17 The input signal to an ideal integrator circuit with $RC = 2 \times 10^{-3}$ s and $V_{cc} = 15$ V is given by $\upsilon_s(t) = 2\sin 100t$ V. What is $\upsilon_{out}(t)$?

Solution:

$$\upsilon_{out}(t) = -\frac{1}{RC}\int_{t_0}^{t} \upsilon_i \, dt + \upsilon_{out}(t_0).$$

Assuming the integration started at $t_0 = 0$ at which time $\upsilon_{out}(0) = 0$,

$$\begin{aligned}\upsilon_{out}(t) &= -\frac{1}{2 \times 10^{-3}}\int_0^t 2\sin 100t \, dt \\ &= \frac{2}{2 \times 10^{-3} \times 100}\cos(100t)\Big|_0^t \\ &= 10[\cos(100t) - 1] \text{ V}.\end{aligned}$$

Exercise 5-18 Repeat Exercise 5-17 for a differentiator instead of an integrator.

Solution:

$$\begin{aligned} v_{\text{out}}(t) &= -RC\,\frac{dv_{\text{i}}}{dt} \\ &= -2\times 10^{-3}\,\frac{d}{dt}[2\sin(100t)] \\ &= -0.4\cos(100t)\ \text{V}. \end{aligned}$$

Exercise 5-19 A CMOS inverter with $C_D^n + C_D^p = 20$ fF has a fall time of 1 ps. What is the value of its gain constant?

Solution:

$$t_{fall} = \frac{C_D^n + C_D^p}{g},$$

$$g = \frac{C_D^n + C_D^p}{t_{fall}} = \frac{20 \times 10^{-15}}{10^{-12}} = 2 \times 10^{-2} \text{ A/V}.$$

Exercise 6-1 For the circuit in Fig. E6-1, determine $v_C(0)$, $i_L(0)$, $v_L(0)$, $i_C(0)$, $v_C(\infty)$, $i_L(\infty)$.

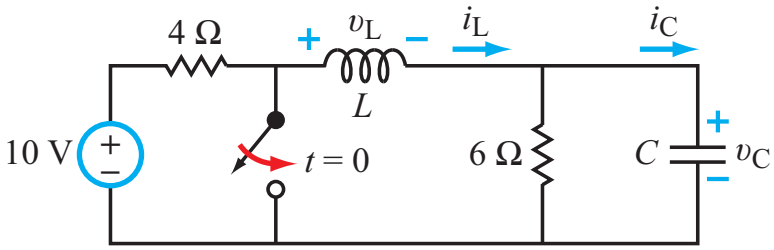
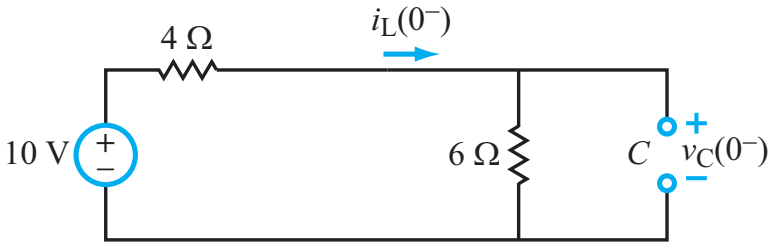


Figure E6.1

Solution:

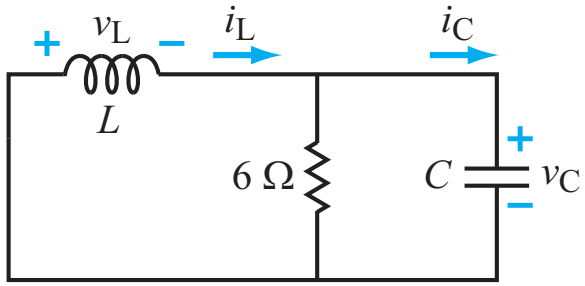
Before $t = 0$:



$$v_C(0) = v_C(0^-) = \frac{6}{4+6}\,10 = 6 \text{ V},$$

$$i_L(0) = i_L(0^-) = \frac{10}{4+6} = 1 \text{ A}.$$

After $t = 0$:



$$v_{\mathrm{L}}(0) = -v_{\mathrm{C}}(0) = -6 \text{ V},$$

$$i_{\mathrm{C}}(0) = i_{\mathrm{L}}(0) - \frac{v_{\mathrm{C}}(0)}{6} = 0 \text{ A},$$

$$v_{\mathrm{C}}(\infty) = 0 \text{ V (no sources and closed loop access to resistors)},$$

$$i_{\mathrm{L}}(\infty) = 0 \text{ A (no sources and closed loop access to resistors)}.$$

Exercise 6-2 For the circuit in Fig. E6-2, determine $v_C(0)$, $i_L(0)$, $v_L(0)$, $i_C(0)$, $v_C(\infty)$, and $i_L(\infty)$.

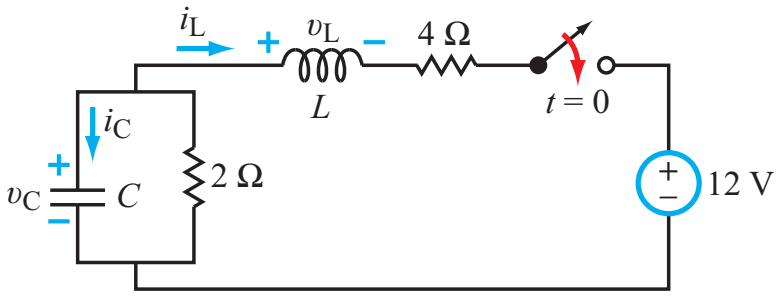
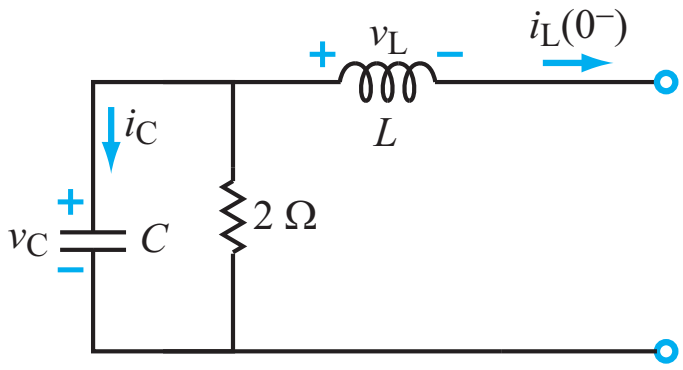


Figure E6.2

Solution:

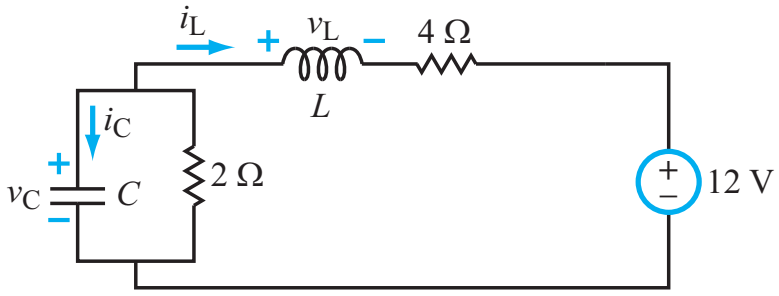
Before $t = 0$:



Hence:

$$v_C(0) = v_C(0^-) = 0 \text{ V (no sources and closed loop access to resistors)},$$
$$i_L(0) = i_L(0^-) = 0 \text{ A}.$$

After $t = 0$:

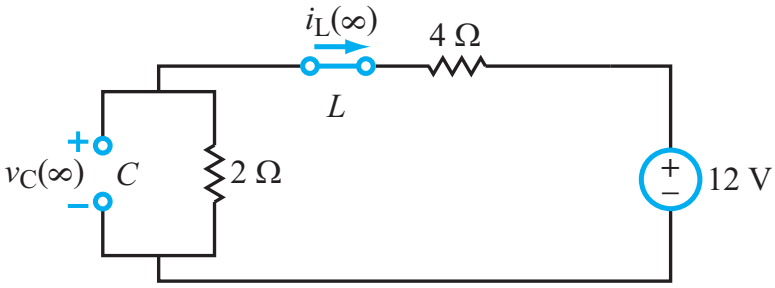


$$v_L(0) = v_C(0) - i_L(0) \times 4 - 12 = -12 \text{ V},$$

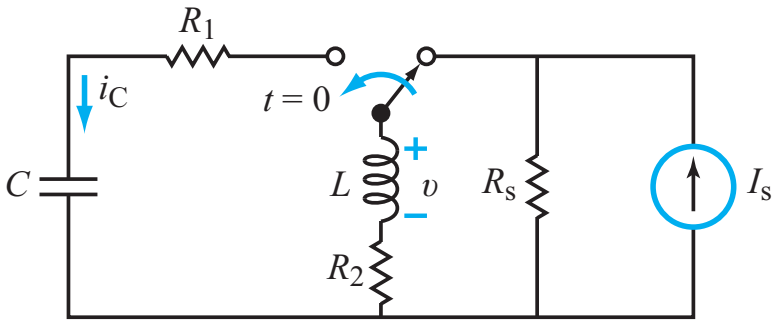
$$i_C(0) = \frac{v_C(0)}{2} = 0 \text{ A},$$

$$v_C(\infty) = \frac{2}{2+4} 12 \text{ V} = 4 \text{ V},$$

$$i_L(\infty) = \frac{v_C(\infty) - 12}{4} = -2 \text{ A}.$$

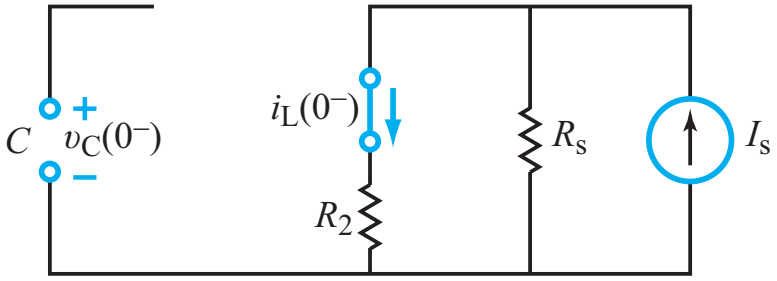


Exercise 6-3 After interchanging the locations of L and C in Fig. 6-9(a), repeat Example 6-4 to determine $v_c(t)$ across C.



Solution:

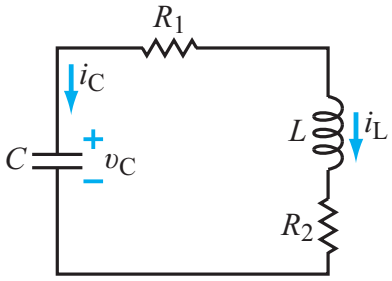
Before $t = 0$:



$$v_C(0) = v_C(0^-) = 0 \text{ V (assume capacitor initially uncharged)},$$

$$i_L(0) = i_L(0^-) = \frac{R_s}{R_2 + R_s} I_s = \frac{10}{0.2 + 10} 2 = 1.961 \text{ A}.$$

After $t = 0$:



$$i_C(0) = -i_L(0) = -1.961 \text{ A},$$
$$v_C'(0) = \frac{i_C(0)}{C} = -\frac{1.961}{5 \times 10^3} = -392.2 \text{ V/s},$$
$$R = R_1 + R_2 = 2.01 \ \Omega.$$

Since R, L, and C are the same as in Example 6-4:

$$\alpha = \frac{R}{2L} = 201 \text{ Np/s},$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = 200 \text{ rad/s},$$
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -181 \text{ Np/s},$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -221 \text{ Np/s}.$$

Apply new initial conditions:

$$v_C(0) = A_1 + A_2 = 0,$$
$$v_C'(0) = s_1 A_1 + s_2 A_2 = -392.2,$$

which leads to

$$A_1 = -\frac{392.2}{s_1 - s_2} = -\frac{392.2}{-181 - (-221)} = -9.79 \text{ V},$$
$$A_2 = -A_1 = 9.79 \text{ V},$$
$$v_C(t) = (A_1 e^{s_1 t} + A_2 e^{s_2 t})\, u(t),$$
$$v_C(t) = 9.79(e^{-221t} - e^{-181t})\, u(t) \ \text{ V}.$$

Exercise 6-4 The switch in Fig. E6.4 is moved to position 2 after it had been in position 1 for a long time. Determine: (a) $v_C(0)$ and $i_C(0)$, and (b) $i_C(t)$ for $t \geq 0$.

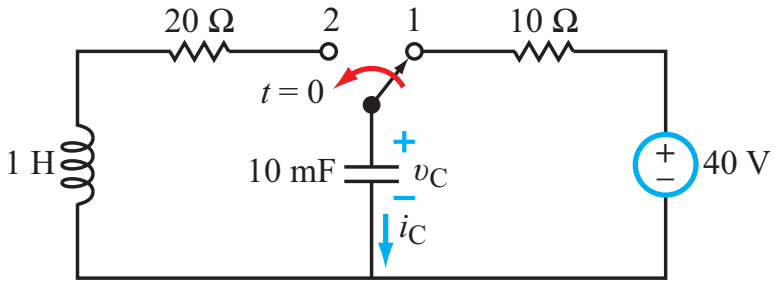
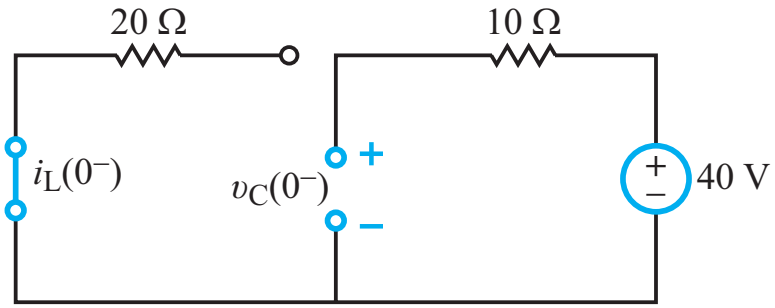


Figure E6.4

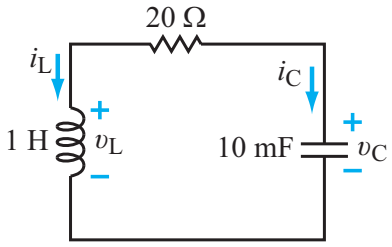
Solution:

Before $t = 0$:



$$v_C(0) = v_C(0^-) = 40 \text{ V},$$
$$i_L(0) = i_L(0^-) = 0 \text{ A}.$$

After $t = 0$:



$$
\begin{aligned}
i_{\mathrm{C}}(0) &= -i_{\mathrm{L}}(0) = 0 \text{ A}, \\
v'_{\mathrm{C}}(0) &= \frac{i_{\mathrm{C}}(0)}{C} = 0, \\
\alpha &= \frac{R}{2L} = \frac{20}{2 \times 1} = 10, \\
\omega_0 &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 0.01}} = 10.
\end{aligned}
$$

Since $\alpha = \omega_0$, the circuit is critically damped. Apply initial conditions:

$$
\begin{aligned}
v_{\mathrm{C}}(t) &= (B_1 + B_2 t) e^{-\alpha t}\, u(t), \\
v'_{\mathrm{C}}(t) &= [-(B_1 + B_2 t)\alpha e^{-\alpha t} + B_2 e^{-\alpha t}]\, u(t), \\
v'_{\mathrm{C}}(t) &= [(1 - \alpha t)B_2 - \alpha B_1] e^{-alt}\, u(t), \\
v_{\mathrm{C}}(0) &= B_1, \\
B_1 &= v_{\mathrm{C}}(0) = 40, \\
v'_{\mathrm{C}}(0) &= B_2 - \alpha B_1, \\
B_2 &= v'_{\mathrm{C}}(0) + \alpha B_1, \\
B_2 &= 0 + 10 \times 40 = 400, \\
i_{\mathrm{C}}(t) &= C v'_{\mathrm{C}}(t), \\
&= C[(1 - \alpha t)B_2 - \alpha B_1] e^{-\alpha t}\, u(t), \\
&= 0.01[(1 - 10t)400 - 10 \times 40] e^{-10t}\, u(t), \\
&= -40 t e^{-10t}\, u(t) \text{ A}.
\end{aligned}
$$

Exercise 6-5 The circuit in Fig. E6.5 is a replica of the circuit in Fig. E6.4, but with the capacitor and inductor interchanged in location. Determine: (a) $i_L(0)$ and $v_L(0)$, and (b) $i_L(t)$ for $t \geq 0$.

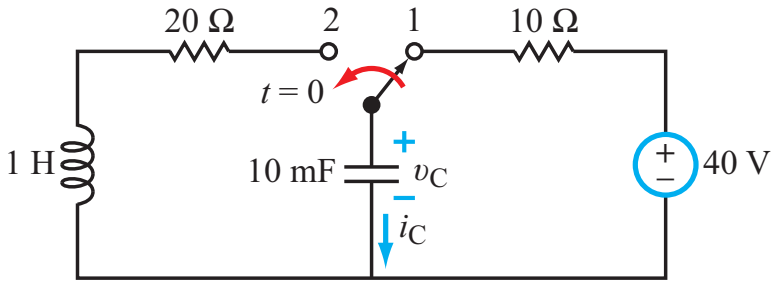
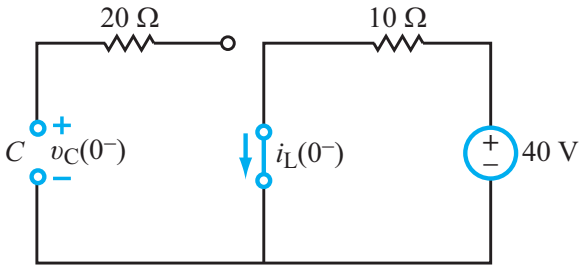


Figure E6.5

Solution:

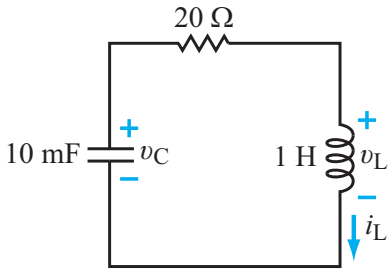
Before $t = 0$:



$$v_C(0) = v_C(0^-) = 0 \text{ V},$$

$$i_L(0) = i_L(0^-) = \frac{40}{10} = 4 \text{ A}.$$

After $t = 0$:



Since the capacitor is initially a short circuit:

$$\upsilon_{\mathrm{L}}(0) = -20 i_{\mathrm{L}}(0) = -20 \times 4 = -80 \text{ V},$$
$$\alpha = \frac{R}{2L} = \frac{20}{2 \times 1} = 10,$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 0.01}} = 10.$$

The circuit is critically damped.

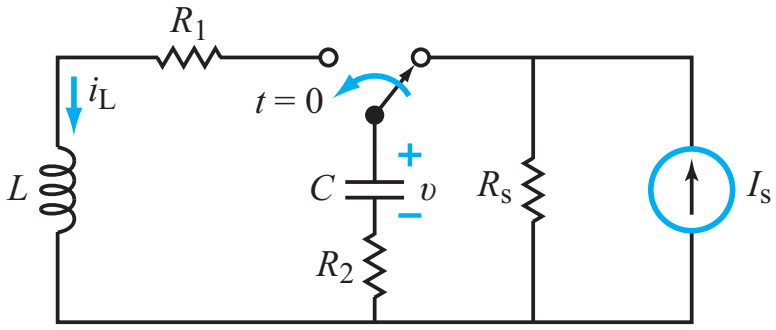
$$i_{\mathrm{L}}(t) = (B_1 + B_2 t) e^{-\alpha t}\, u(t),$$
$$i'_{\mathrm{L}}(t) = [(1 - \alpha t) B_2 - \alpha B_1] e^{-\alpha t}\, u(t),$$
$$B_1 = i_{\mathrm{L}}(0) = 4,$$
$$B_2 = i'_{\mathrm{L}}(0) + \alpha B_1 = \frac{\upsilon_{\mathrm{L}}(0)}{L} + \alpha B_1,$$
$$= -\frac{80}{1} + 10 \times 4 = -40.$$

Hence

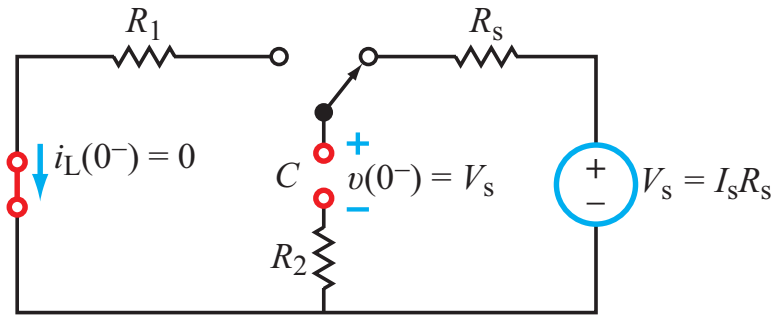
$$i_{\mathrm{L}}(t) = 4(1 - 10t) e^{-10t}\, u(t) \text{ A}.$$

Exercise 6-6 Repeat Example 6-4 after replacing the 8 V source with a short circuit and changing the value of R_1 to 1.7 Ω.

Solution:



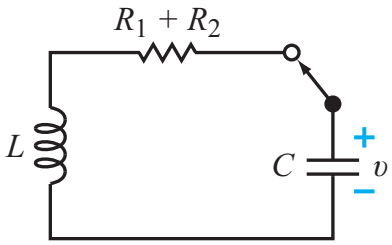
Before $t = 0$:



where we used source transformation on (I_s, R_s). From the circuit

$$v(0) = v(0^-) = I_s R_s = 20 \text{ V},$$
$$i_L(0) = i_L(0^-) = 0 \text{ A}.$$

After $t = 0$:



$$
\begin{aligned}
v'(0) &= \frac{i_C(0)}{C} = -\frac{i_L(0)}{C} = 0, \\
\alpha &= \frac{R}{2L} = \frac{R_1 + R_2}{2L} = \frac{1.7 + 0.2}{2 \times 0.005} = 190, \\
\omega_0 &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.005 \times 0.005}} = 200, \\
\omega_d &= \sqrt{\omega_0^2 - \alpha^2} = \sqrt{200^2 - 190^2} = 62.45, \\
v(t) &= (D_1 \cos \omega_d t + D_2 \sin \omega_d t) e^{-\alpha t}\, u(t), \\
v(0) &= D_1, \\
D_1 &= v(0) = 20, \\
D_2 &= \frac{\alpha\, v(0)}{\omega_d} = \frac{190 \times 20}{62.45} = 60.85, \\
v(t) &= (20 \cos 62.45t + 60.85 \sin 62.45t) e^{-190t}\, u(t) \ \text{ V}.
\end{aligned}
$$

Exercise 6-7 Determine the initial and final values for i_L in the circuit of Fig. E6.7, and provide an expression for $i_L(t)$.

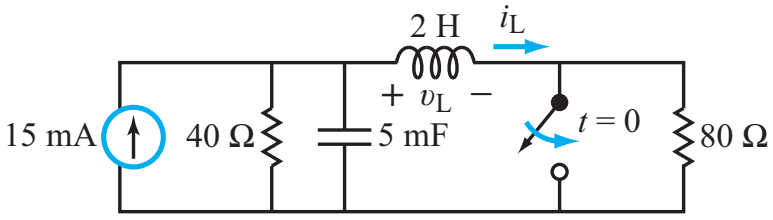
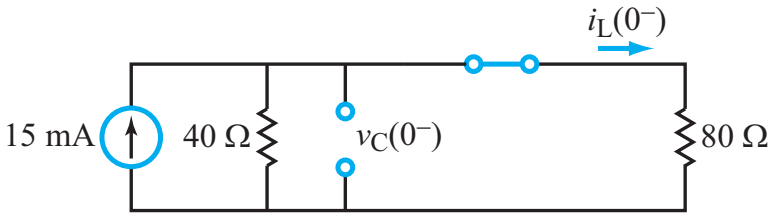


Figure E6.7

Solution:

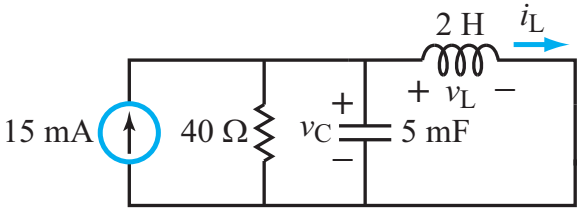
Before $t = 0$:



$$v_C(0) = 0.015(40\ \Omega \parallel 80\ \Omega) = 0.015\ \frac{40 \times 80}{40 + 80} = 0.4,$$

$$i_L(0) = \frac{v_C(0)}{80} = \frac{0.4}{80} = 0.005.$$

After $t = 0$:



$$i_{\mathrm{L}}'(0) = \frac{v_{\mathrm{L}}(0)}{L} = \frac{v_{\mathrm{C}}(0)}{L} = \frac{0.4}{2} = 0.2,$$

$$i_{\mathrm{L}}(\infty) = 0.015 \quad (L \text{ acts like a short circuit at } t = \infty)$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 40 \times 0.005} = 2.5,$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 0.005}} = 10 \text{ rad/s},$$

$$\omega_{\mathrm{d}} = \sqrt{\omega_0^2 - \alpha^2} = 9.68 \text{ rad/s}.$$

$$i_{\mathrm{L}}(t) = [i_{\mathrm{L}}(\infty) + e^{-\alpha t}(D_1 \cos \omega_{\mathrm{d}} t + D_2 \cos \omega_{\mathrm{d}} t)],$$

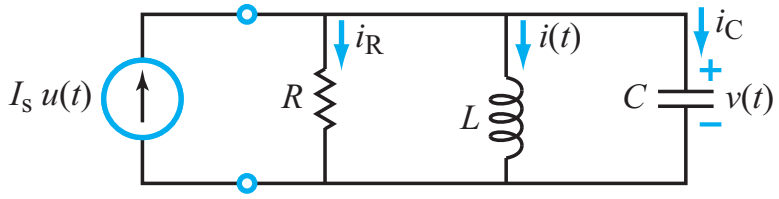
$$D_1 = i_{\mathrm{L}}(0) - i_{\mathrm{L}}(\infty) = 0.005 - 0.015 = -0.010,$$

$$D_2 = \frac{i_{\mathrm{L}}' + \alpha[i_{\mathrm{L}}(0) - i_{\mathrm{L}}(\infty)]}{\omega_{\mathrm{d}}} = 0.01808.$$

Hence,

$$i_{\mathrm{L}}(t) = \left\{15 - [10\cos 9.68t - 18.08 \sin 9.68t]e^{-2.5t}\right\} \text{ mA}.$$

Exercise 6-8 In the parallel RLC circuit shown in Fig. 6-13(b), how much energy will be stored in L and C at $t = \infty$?



Solution: At $t = \infty$, L is a short circuit:

$$v_C(\infty) = 0,$$
$$w_C(\infty) = 0,$$
$$i_L(\infty) = I_s,$$
$$w_L(\infty) = \frac{1}{2} L\, i_L^2(\infty) = \frac{1}{2} L I_s^2.$$

Exercise 6.9 Develop an expression for $i_C(t)$ in the circuit of Fig. E6.14 for $t \geq 0$.

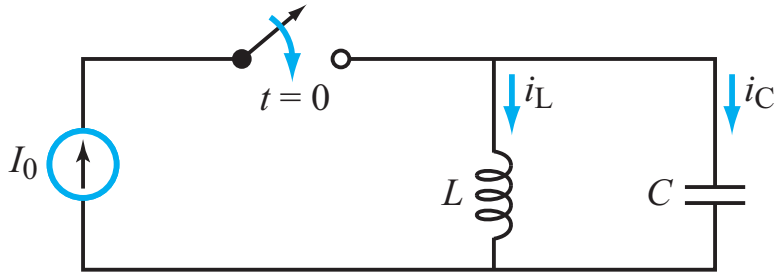


Figure E6.9

Solution:

Before $t = 0$:

$$v_C(0) = 0,$$
$$i_L(0) = 0.$$

After $t = 0$:

$$i'_L(0) = \frac{v_L(0)}{L} = \frac{v_C(0)}{L} = 0,$$
$$i_L(\infty) = I_0,$$
$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times \infty \times C} = 0,$$
$$\omega_0 = \frac{1}{\sqrt{LC}}.$$

Since α is less than ω_0, the circuit is underdamped:

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \omega_0,$$
$$D_1 = i_L(0) - i_L(\infty) = 0 - I_0 = -I_0,$$
$$D_2 = \frac{i'_L(0) + \alpha D_1}{\omega_d} = \frac{0 - 0 \times I_0}{\omega_0} = 0,$$
$$i_L(t) = i_L(\infty) + [D_1 \cos \omega_d t + D_2 \sin \omega_d t]e^{-\alpha t},$$
$$i_L(t) = I_0 - I_0 \cos \omega_0 t = I_0(1 - \cos \omega_0 t),$$
$$i_C(t) = I_0 - i_L(t) = I_0 - (I_0 - I_0 \cos \omega_0 t)$$
$$= I_0 \cos \omega_0 t.$$

Hence, without a resistor in the circuit, the circuit behaves like an oscillator.

Exercise 6.10 For the circuit in Fig. E6.10, determine $i_C(t)$ for $t \geq 0$.

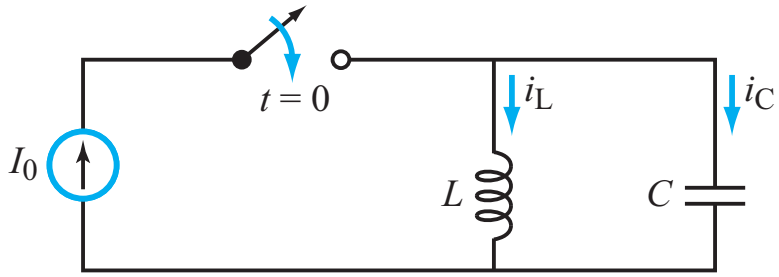


Figure E6.10

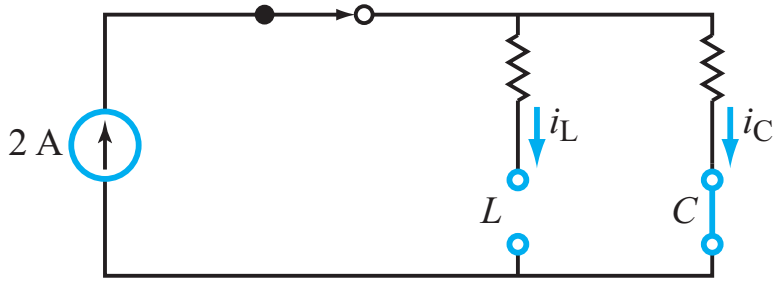
Solution:

Before $t = 0$, there are no sources:

$$v_C(0) = 0,$$
$$i_L(0) = 0.$$

At $t = 0$**:**

$$i_C(0) = 2 - i_L(0) = 2,$$
$$i'_L(0) = \frac{v_L(0)}{L} = \frac{R\, i_C(0)}{L} = \frac{3 \times 2}{2} = 3.$$



After $t = 0$:

$$
\begin{aligned}
&R\, i_{\mathrm{L}}(t) + L\,\frac{di_{\mathrm{L}}}{dt} = R\, i_{\mathrm{C}}(t) + v_{\mathrm{C}}(t),\\
&i_{\mathrm{L}}(t) = 2 - i_{\mathrm{C}}(t),\\
&\frac{di_{\mathrm{L}}}{dt} = -\frac{di_{\mathrm{C}}}{dt}\,,\\
&R[2 - i_{\mathrm{C}}(t)] - L\,\frac{di_{\mathrm{C}}}{dt} = R\, i_{\mathrm{C}}(t) + \upsilon_{\mathrm{C}}(t),\\
&2R - L\,\frac{di_{\mathrm{C}}}{dt} = 2R\, i_{\mathrm{C}}(t) = \upsilon_{\mathrm{C}}(t),\\
&-LC\,\frac{d^2 i_{\mathrm{C}}}{dt^2} = 2RC\,\frac{di_{\mathrm{C}}}{dt} + C\,\frac{d\upsilon_{\mathrm{C}}}{dt}\,,\\
&\frac{d^2 i_{\mathrm{C}}}{dt^2} + \frac{2R}{L}\,\frac{di_{\mathrm{C}}}{dt} + \frac{1}{LC}\,i_{\mathrm{C}}(t) = 0,\\
&i''_{\mathrm{C}} + \frac{2R}{L}\,i'_{\mathrm{C}} + \frac{1}{LC}\,i_{\mathrm{C}} = 0,\\
&i'_{\mathrm{C}}(0) = -i'_{\mathrm{L}}(0) = -3,\\
&i_{\mathrm{C}}(\infty) = 0,\\
&a = \frac{2R}{L} = \frac{2\times 3}{2} = 3,\\
&b = \frac{1}{LC} = \frac{1}{2\times 0.02} = 25,\\
&\alpha = \frac{a}{2} = 1.5,\\
&\omega_0 = \sqrt{b} = 5.
\end{aligned}
$$

The circuit is underdamped:

$$
\begin{aligned}
\omega_{\mathrm{d}} &= \sqrt{\omega_0^2 - \alpha^2} = \sqrt{25 - 1.5^2} = 4.77,\\
D_1 &= i_{\mathrm{C}}(0) - i_{\mathrm{C}}(\infty) = 2 - 0 = 2,\\
D_2 &= \frac{i'_{\mathrm{C}}(0) + \alpha D_1}{\omega_{\mathrm{d}}} = \frac{-3 + 1.5\times 2}{4.77} = 0,\\
i_{\mathrm{C}}(t) &= [i_{\mathrm{C}}(\infty) + (D_1\cos\omega_{\mathrm{d}}t + D_2\sin\omega_{\mathrm{d}}t)e^{-\alpha t}]\; u(t),\\
i_{\mathrm{C}}(t) &= (2e^{-\alpha t}\cos 4.77t)\; u(t)\;\ \text{A}.
\end{aligned}
$$

Exercise 6.11 Given the component values in the Multisim circuit of Fig. 6-18, what are the values of ω_0 and α for the circuit response?

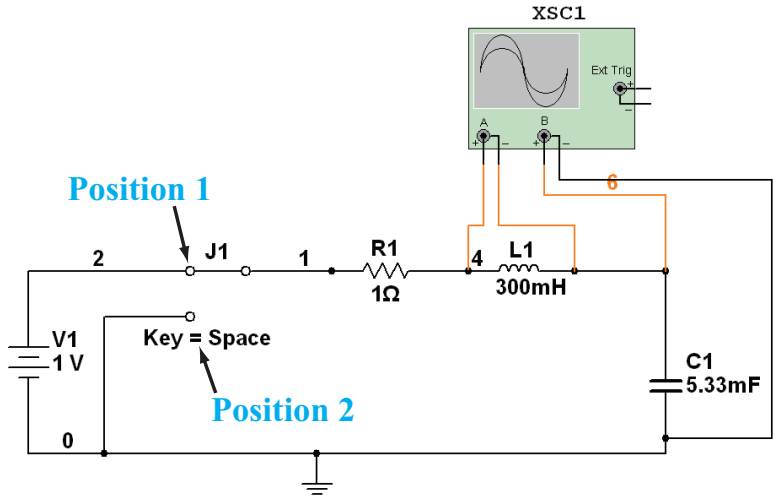
Solution:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.3 \times 5.33 \times 10^{-3}}} = 25.0 \text{ rad/s},$$

$$\alpha = \frac{R}{2L} = \frac{1}{2 \times 0.3} = 1.67 \text{ Np/s},$$

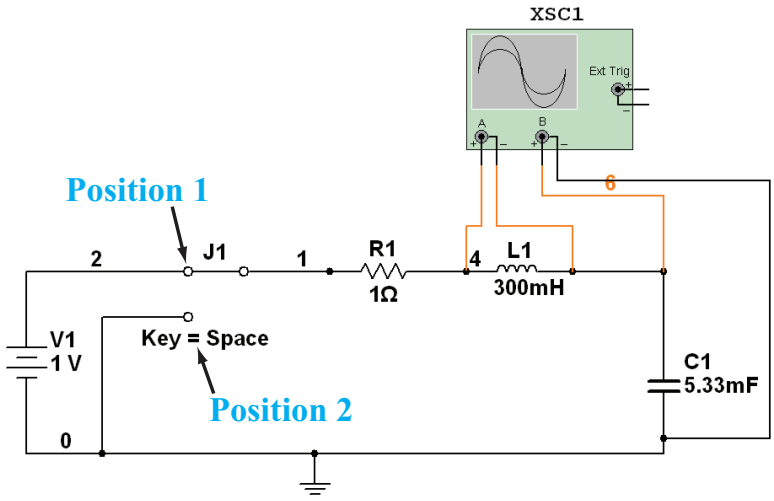
$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{25.0^2 - 1.67^2} = 24.95 \text{ rad/s}.$$

Exercise 6-12 Is the natural response for the circuit in Fig. 6-18 over-, under-, or critically damped? You can determine this both graphically (from the oscilloscope) and mathematically, by comparing ω_0 and α.



Solution: $\alpha < \omega_0$, so the circuit is underdamped, as is evident in the circuit's oscillatory response.

Exercise 6-13 Modify the value of R in the circuit of Fig. 6-18 so as to obtain a critically damped response.



Solution: For a critically damped circuit, $\alpha = \omega_0$:

$$\alpha = \frac{R}{2L} = \omega_0 = 25.0,$$

$$\frac{R}{2 \times 0.3} = 25.0,$$

$$R = 15.0\ \Omega.$$

Exercise 6-14 Calculate ω_0, α, and ω_d for the RLC circuit in Fig. 6-22. How do ω_0 and ω_d compare with the angular frequency of the current source? This result, as we will learn later when we study resonant circuits in Chapter 9, is not at all by coincidence.

Solution:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 10^{-6}}} = 31.6 \times 10^3,$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 10^3 \times 10^{-6}} = 500,$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{(31.6 \times 10^3)^2 - 500^2} = 31.6 \times 10^3 \text{ rad/s}.$$

The angular frequency of the current source is:

$$\omega_{src} = 2\pi f_{src} = 2\pi \times 5033 = 31.6 \times 10^3 \text{ rad/s},$$

which is the same as ω_0 and ω_d.

Exercise 6-15 Ideally, we would like the response of the RFID tag to take a very long time to decay down to zero, so as to contain as many digital bits as possible. What determines the decay time? Change the values of some of the components in Fig. 6-22 so as to decrease the damping coefficient by a factor of 2.

Solution: The decay time of the RFID is determined by the damping coefficient α. To reduce α by a factor of two:

$$\alpha = \frac{1}{2RC} = \frac{500}{2} = 250.$$

Leave C unchanged at 1 μF and solve for R:

$$\frac{1}{2R \times 10^{-6}} = 250,$$

or

$$R = 2000\ \Omega.$$

Exercise 7-1 Provide an expression for a 100-V, 60-Hz voltage that exhibits a minimum at $t = 0$.

Solution:

$$\begin{aligned} v(t) &= A\cos(2\pi ft + \phi), \\ A &= 100 \text{ V (given)}, \\ f &= 60 \text{ Hz (given)}. \end{aligned}$$

At $t = 0$, $v(t)$ is a minimum. Hence,

$$v(0) = -A = A\cos\phi_1 \quad \Longrightarrow \quad \phi = 180^\circ,$$

and

$$v(t) = 100\cos(120\pi + 180^\circ) \text{ V}.$$

Exercise 7-2 Given two current waveforms:

$$i_1(t) = 3\cos \omega t,$$
$$i_2(t) = 3\sin(\omega t + 36^\circ),$$

does $i_2(t)$ lead or lag $i_1(t)$ and by what phase angle?

Solution:

$$\begin{aligned} i_1(t) &= 3\cos \omega t, \\ i_2(t) &= 3\sin(\omega t + 36^\circ) \\ &= 3\cos(90^\circ - \omega t - 36^\circ) \\ &= 3\cos(\omega t + 36 - 90^\circ) \\ &= 3\cos(\omega t - 54^\circ). \end{aligned}$$

Since $\phi_2 < \phi_1$, $i_2(t)$ lags $i_1(t)$ by 54°.

Exercise 7-3 Express the following complex functions in polar form:

$$\mathbf{z}_1 = (4 - j3)^2,$$
$$\mathbf{z}_2 = (4 - j3)^{1/2}.$$

Solution:

$$\begin{aligned}
\mathbf{z}_1 &= (4 - j3)^2 \\
&= [\sqrt[+]{4^2 + 3^2}\, e^{-j\tan^{-1} 3/4}]^2 = (5e^{-j36.87^\circ})^2 = 25e^{-j73.74^\circ} \\
\mathbf{Z}_2 &= (4 - j3)^{1/2} \\
&= \left[\sqrt[+]{4^2 + 3^2}\, e^{-j\tan^{-1} 3/4}\right]^{1/2} \\
&= \pm\sqrt{5}\, e^{-j18.43^\circ}.
\end{aligned}$$

Exercise 7-4 Show that $\sqrt{2j} = \pm(1+j)$.

Solution:

$$\begin{aligned}\sqrt{2j} &= \sqrt{2e^{j90^\circ}} \\ &= \pm\sqrt{2}\, e^{j45^\circ} \\ &= \pm\sqrt{2}\left(\frac{\cos 45^\circ + j\sin 45^\circ}{2}\right) \\ &= \pm\sqrt{2}\left(\frac{\sqrt{2}+j\sqrt{2}}{2}\right) = \pm(1+j).\end{aligned}$$

Exercise 7-5 Determine the phasor counterparts of the following waveforms:

(a) $i_1(t) = 2\sin(6\times 10^3 t - 30^\circ)$ A,

(b) $i_2(t) = -4\sin(1000t + 136^\circ)$ A.

Solution:

$$\begin{aligned} i_1(t) &= 2\sin(6\times 10^3 t - 30^\circ)\ \text{A} \\ &= 2\cos(6\times 10^3 t - 30^\circ - 90^\circ)\ \text{A}. \end{aligned}$$

Hence, $\phi_1 = -120^\circ$.

$$\mathbf{I}_1 = 2\angle \underline{-120^\circ}\ \text{A},$$

$$\begin{aligned} i_2(t) &= -4\sin(1000t + 136^\circ)\ \text{A} \\ &= 4\sin(1000t + 136^\circ - 180^\circ)\ \text{A} \\ &= 4\cos(1000t + 136^\circ - 180^\circ - 90^\circ)\ \text{A} \\ &= 4\cos(1000t - 134^\circ)\ \text{A}. \end{aligned}$$

Hence $\phi_2 = -134^\circ$,

$$\mathbf{I}_2 = 4\angle \underline{-134^\circ}\ \text{A}.$$

Exercise 7-6 Obtain the time-domain waveforms (in standard cosine format) corresponding to the following phasors, at angular frequency $\omega = 3 \times 10^4$ rad/s:

(a) $\mathbf{V}_1 = (-3 + j4)$ V,

(b) $\mathbf{V}_2 = (3 - j4)$ V.

Solution:

(a)

$$\begin{aligned}\mathbf{V}_1 &= (-3 + j4)\ \text{V} \\ &= \sqrt{3^2 + 4^2}\, e^{j126.87^\circ} \qquad \text{(second quadrant)}, \\ v_1(t) &= 5\cos(3 \times 10^4 t + 126.87^\circ)\ \text{V}.\end{aligned}$$

(b)

$$\begin{aligned}\mathbf{V}_2 &= (3 - j4)\ \text{V} \\ &= \sqrt{3^2 + 4^2}\, e^{j(-53.13^\circ)} \qquad \text{(fourth quadrant)}, \\ v_2(t) &= 5\cos(3 \times 10^4 t - 53.13^\circ)\ \text{V}.\end{aligned}$$

Exercise 7-7 At $\omega = 10^6$ rad/s, the phasor voltage across and current through a certain element are given by: $\mathbf{V} = 4\angle -20^\circ$ V and $\mathbf{I} = 2\angle 70^\circ$ A. What type of element is it?

Solution:

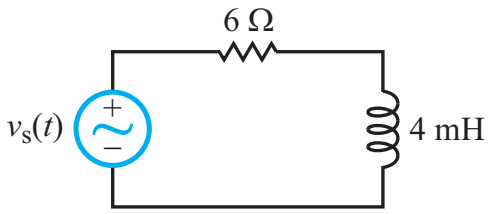
$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{4e^{-j20^\circ}}{2e^{j70^\circ}} = 2e^{-j90^\circ} = -j2\ \Omega.$$

Hence, it is a capacitor and from

$$-j2 = \frac{-j}{\omega C},$$

$$C = \frac{1}{2\omega} = 0.5\ \mu\text{F}.$$

Exercise 7-8 Repeat the analysis of the circuit in Example 7-4 for $v_s(t) = 20\cos(2\times10^3 t + 60^\circ)$ V, $R = 6\ \Omega$, and $L = 4$ mH.



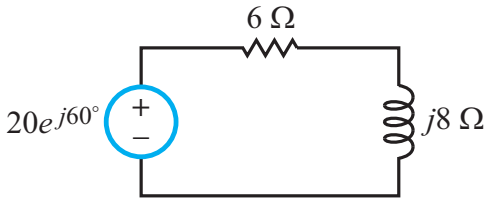
Solution:

Step 1:

$$v_s(t) = 20\cos(2\times10^3 t + 60^\circ),$$
$$\mathbf{V}_s = 20e^{j60^\circ}.$$

Step 2: Circuit to Phasor Domain

$$\mathbf{Z}_L = j\omega L = j2\times10^3\times4\times10^{-3} = j8\ \Omega.$$



Step 3: KVL in Phasor Domain

$$R\mathbf{I} + j\omega L\mathbf{I} = \mathbf{V}_s,$$
$$6\mathbf{I} + j8\mathbf{I} = 20e^{j60^\circ}.$$

Step 4: Solve for Unknown Variable

$$\mathbf{I} = \frac{20e^{j60^\circ}}{6+j8} = \frac{20e^{j60^\circ}}{10e^{j53.13^\circ}} = 2\angle\underline{6.9^\circ}\ \text{A}.$$

Find Inductor Voltage:

$$\mathbf{V}_L = j\omega L\mathbf{I} = (j8)(2\angle\underline{6.9^\circ})$$
$$= 16\angle\underline{96.9^\circ}\ \text{V}.$$

Step 5: Convert Back to Time Domain

$$\begin{aligned} v_{\mathrm{L}}(t) &= \mathfrak{Re}[\mathbf{V}_{\mathrm{L}} e^{j\omega t}] \\ &= \mathfrak{Re}[16 e^{j96.9^\circ} e^{j2\times 10^3 t}]. \end{aligned}$$

$$v_{\mathrm{L}}(t) = 16\cos(2\times 10^3 t + 96.9^\circ)\ \mathrm{V}.$$

Exercise 7-9 Determine the input impedance at $\omega = 10^5$ rad/s for each of the circuits in Fig. E7-9.

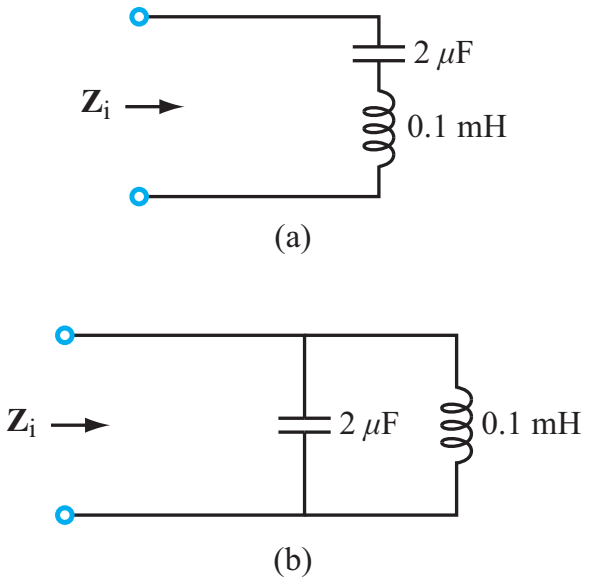


Figure E7.9

Solution:
(a)

$$\begin{aligned}\mathbf{Z}_i &= \mathbf{Z}_L + \mathbf{Z}_C \\ &= j\omega L - \frac{j}{\omega C} \\ &= j \times 10^5 \times (0.1 \times 10^{-3}) - \frac{j}{(10^5)(2 \times 10^{-6})} \\ &= j5\ \Omega.\end{aligned}$$

(b)

$$
\begin{aligned}
\mathbf{Z}_{\mathrm{i}} &= \left(\frac{1}{\mathbf{Z}_{\mathrm{C}}} + \frac{1}{\mathbf{Z}_{\mathrm{L}}}\right)^{-1} \\
&= \left(j\omega C + \frac{1}{j\omega L}\right)^{-1} \\
&= \left[\frac{-(10^5)(2\times 10^{-6})}{j} + \frac{1}{j(10^5)(0.1\times 10^{-3})}\right]^{-1} \\
&= \left[\frac{-2+1}{j(10^5)(0.1\times 10^{-3})}\right]^{-1} \\
&= \left[\frac{-1}{j(10^5)(0.1\times 10^{-3})}\right]^{-1} \\
&= -j10\ \Omega.
\end{aligned}
$$

Exercise 7-10 Convert the Y-impedance circuit in Fig. E7-10 into a Δ-impedance circuit.

Figure E7-10

Solution:

$$\mathbf{Z}_1 = j5\ \Omega,$$
$$\mathbf{Z}_2 = j5\ \Omega,$$
$$\mathbf{Z}_3 = -j10\ \Omega,$$
$$\begin{aligned}\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3 &= (j5)(j5) + (j5)(-j10) + (j5)(-j10)\\ &= j^2 25 - 2 \times j^2 50\\ &= -25 + 100\\ &= 75\ (\Omega)^2.\end{aligned}$$

From Eq. (7.86a),

$$\begin{aligned}\mathbf{Z}_a &= \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3}{\mathbf{Z}_1}\\ &= \frac{75}{j5} = -j15\ \Omega.\end{aligned}$$

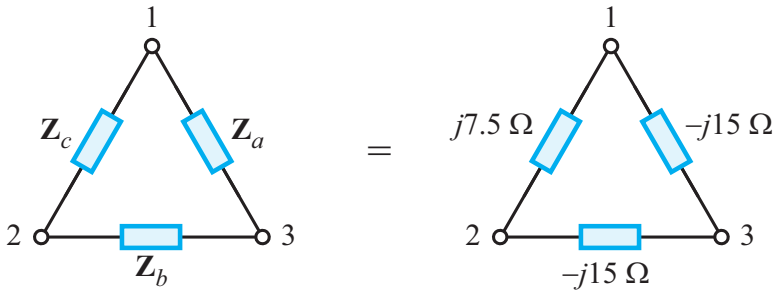
From Eq. (7.86b),

$$\begin{aligned}\mathbf{Z}_b &= \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3}{\mathbf{Z}_2}\\ &= \frac{75}{j5} = -j15\ \Omega.\end{aligned}$$

From Eq. (7.86c),

$$\begin{aligned}\mathbf{Z}_c &= \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3}{\mathbf{Z}_3}\\ &= \frac{75}{-j10} = -j7.5\ \Omega.\end{aligned}$$

So


1
$\mathbf{Z}_c$
$\mathbf{Z}_a$
2
3
$\mathbf{Z}_b$
=
1
$j7.5\ \Omega$
$-j15\ \Omega$
2
3
$-j15\ \Omega$

Exercise 7-11 Determine $\mathbf{V}_{\text{Th}}$ and $\mathbf{Z}_{\text{Th}}$ for the circuit in Fig. E7-11 at terminals (a,b).

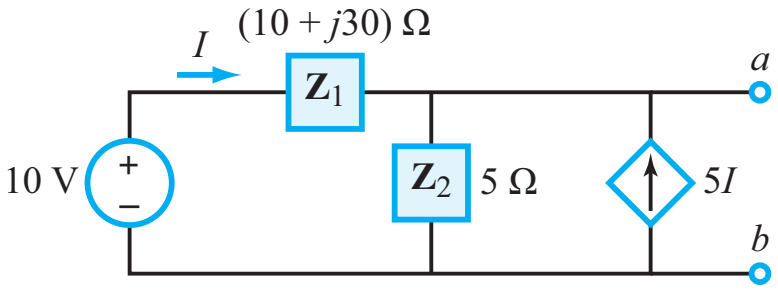


Figure E7-11

Solution:

$\mathbf{V}_{\text{Th}}$:

KCL at $\mathbf{V}_a$:

$$-\mathbf{I}+\frac{\mathbf{V}_a}{5}-5\mathbf{I}=0,$$

$$6\mathbf{I}=\frac{\mathbf{V}_a}{5}.$$

Also,

$$\mathbf{I}=\frac{10-\mathbf{V}_a}{10+j30}.$$

Hence,

$$6\left(\frac{10-\mathbf{V}_a}{10+j30}\right)=\frac{\mathbf{V}_a}{5},$$

$$\frac{60}{10+j30}=\mathbf{V}_a\left(\frac{6}{10+j30}+\frac{1}{5}\right).$$

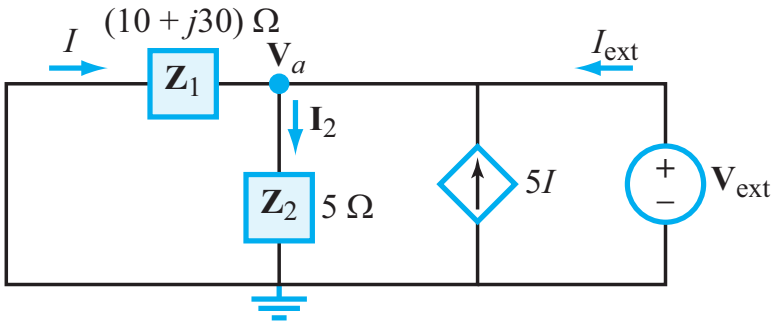
Simplifying leads to

$$\mathbf{V}_a=\frac{60}{8+j6}$$

$$\mathbf{V}_{\text{Th}}=\mathbf{V}_a=6\angle\underline{-36.9^\circ}\ \text{V}.$$

R_{Th}:

Remove source and add external source $\mathbf{V}_{\text{ext}}$:



At node $\mathbf{V}_a$:

$$-\mathbf{I}+\mathbf{I}_2-5\mathbf{I}-\mathbf{I}_{\text{ext}}=0.$$

Also,

$$\mathbf{I}_2=\frac{\mathbf{V}_{\text{ext}}}{\mathbf{Z}_2}=\frac{\mathbf{V}_{\text{ext}}}{5},$$
$$\mathbf{I}=\frac{\mathbf{V}_{\text{ext}}}{\mathbf{Z}_1}=\frac{\mathbf{V}_{\text{ext}}}{10+j30}.$$

Substitution and simplification lead to

$$\mathbf{I}_{\text{ext}}=(0.26-j0.18)\mathbf{V}_{\text{ext}}.$$

Hence,

$$R_{\text{Th}}=\frac{\mathbf{V}_{\text{ext}}}{\mathbf{I}_{\text{ext}}}=\frac{1}{0.26-j0.18}=(2.6+j1.8)\ \Omega.$$

Exercise 7-12 Establish the relative phasor diagram for the circuit in Fig. E7-12 with **V** as the reference phasor.

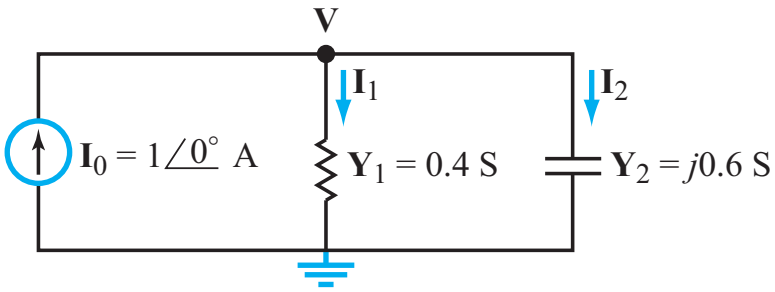
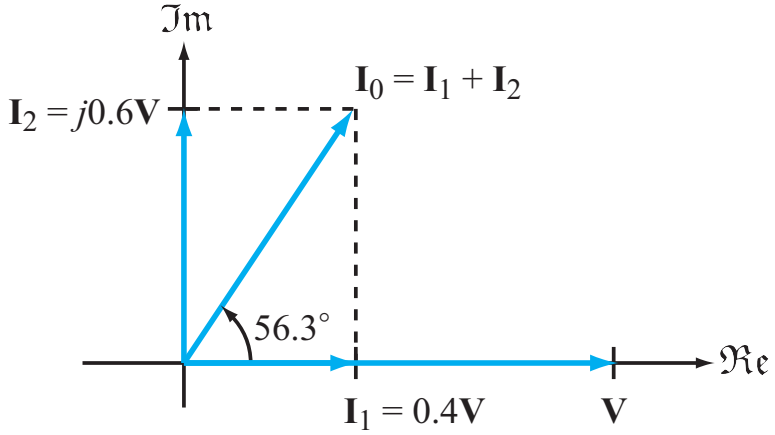


Figure E7.12

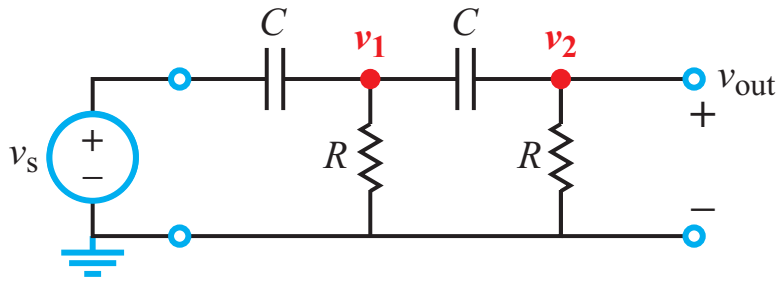
Solution:

$$\begin{aligned}
\mathbf{I}_1 &= \mathbf{V}Y_1 \\
&= 0.4\mathbf{V}, \\
\mathbf{I}_2 &= \mathbf{V}Y_2 \\
&= j0.6\mathbf{V}, \\
\mathbf{I}_0 &= \mathbf{I}_1 + \mathbf{I}_2 \\
&= 0.4\mathbf{V} + j0.6\mathbf{V} \\
&= (0.721\angle 56.3^\circ)\mathbf{V}.
\end{aligned}$$



Exercise 7-13 Repeat Example 7-11, but use only two stages of RC phase shifters.

Solution:



At node $\mathbf{V}_1$ in phasor domain:

$$\frac{\mathbf{V}_1 - \mathbf{V}_s}{\mathbf{Z}_C} + \frac{\mathbf{V}_1}{R} + \frac{\mathbf{V}_1}{\mathbf{Z}_C + R} = 0.$$

Also,

$$\mathbf{V}_{out} = \mathbf{V}_2 = \mathbf{V}_1 \frac{R}{\mathbf{Z}_C + R}.$$

Substitution and simplification leads to

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_s} = \frac{R^4\omega^4C^4 - R^2\omega^2C^2 + j3\omega^3R^3C^3}{(R^4\omega^4C^4 - 1)^2 + 9\omega^2R^2C^2}.$$

Hence

$$\phi_2 = -\tan^{-1}\left(\frac{3\omega^3R^3C^3}{R^4\omega^4C^4 - R^2\omega^2C^2}\right)$$
$$= -\tan^{-3}\left(\frac{3\omega RC}{\omega^2R^2C^2 - 1}\right).$$

For $\phi_2 = 120^\circ$, $\omega = 10^3$ rad/s, and $C = 1\ \mu$F, solution for R gives

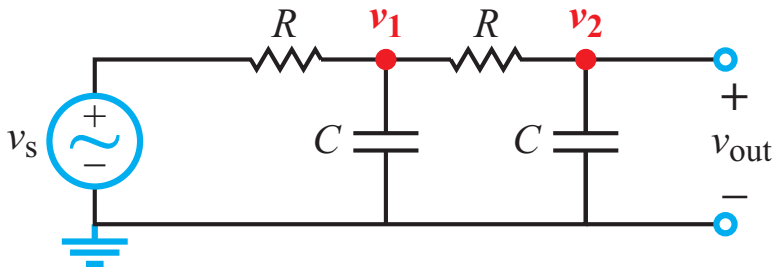
$$R = 2.189\ \text{k}\Omega \simeq 2.2\ \text{k}\Omega.$$

[Mathematically, we get a second solution, namely $R = -456.8$, which we reject.] Using $R = 2.2\ \text{k}\Omega$ in the expression for $\mathbf{V}_{out}/\mathbf{V}_s$ gives

$$\left|\frac{\mathbf{V}_{out}}{\mathbf{V}_s}\right| = 0.634.$$

Exercise 7-14 Design a two-stage RC phase shifter that provides a phase shift of negative 120° at $\omega = 10^4$ rad/s. Assume $C = 1\ \mu$F.

Solution: For negative phase shift, flip capacitor and inductor positions.



KVL at node 1:

$$\frac{\mathbf{V}_1 - \mathbf{V}_s}{R} + \frac{\mathbf{V}_1}{\mathbf{Z}_C} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{R} = 0. \tag{1}$$

Also,

$$\mathbf{V}_{out} = \mathbf{V}_2 = \mathbf{V}_1 \frac{\mathbf{Z}_C}{\mathbf{Z}_C + R}.$$

Solution leads to

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_s} = \frac{-(-1 + R^2\omega^2C^2 + j3\omega RC)}{(-1 + R^2\omega^2)^2 + 9\omega^2R^2C^2},$$

and

$$\phi_2 = \tan^{-1}\left(\frac{3\omega RC}{\omega^2R^2C^2 - 1}\right).$$

For $\phi_2 = -120°$,

$$\tan(-120°) = 1.732 = \left(\frac{3\omega RC}{\omega^2R^2C^2 - 1}\right).$$

For $\omega = 10^4$ rad/s and $C = 1\ \mu$F,

$$R = 218.89\ \Omega \qquad \text{or } R = -45.68\ \Omega.$$

Negative resistance solution is rejected. Hence,

$$R \simeq 220\ \Omega.$$

Exercise 7-15 Write down the node-voltage matrix equation for the circuit in Fig. E7-15.

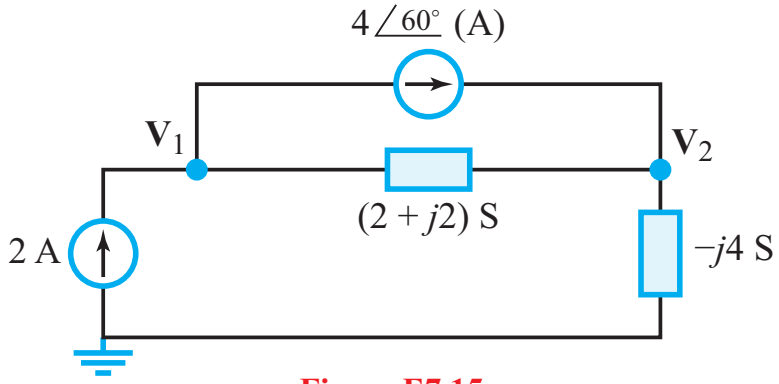


Figure E7.15

Solution: KCL at node $\mathbf{V}_1$:

$$-2+4\angle 60^\circ+(\mathbf{V}_1-\mathbf{V}_2)(2+j2)=0,$$
$$\mathbf{V}_1(2+j2)+\mathbf{V}_2[-(2+j2)]=2-4\angle 60^\circ. \tag{1}$$

KCL at node $\mathbf{V}_1$:

$$-4\angle 60^\circ+(\mathbf{V}_2-\mathbf{V}_1)(2+j2)+\mathbf{V}_2(-j4)=0,$$
$$\mathbf{V}_1[-(2+j2)]+\mathbf{V}_2(2-j2)=4\angle 60^\circ. \tag{2}$$

Put (1) and (2) into matrix form:

$$\begin{bmatrix}(2+j2) & -(2+j2)\\ -(2+j2) & (2-j2)\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}=\begin{bmatrix}2-4e^{j60^\circ}\\ 4e^{j60^\circ}\end{bmatrix}.$$

Alternatively, direct application of Eq. (7.102) leads to the same matrix equation.

Exercise 7-16 Write down the mesh-current matrix equation for the circuit in Fig. E7-16.

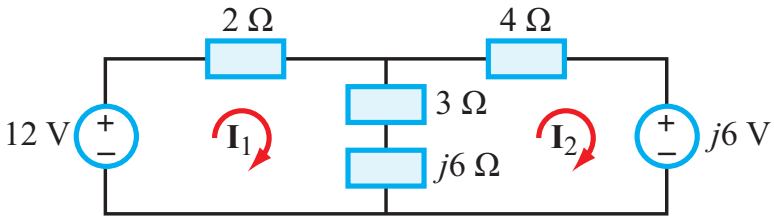


Figure E7-16

Solution: For mesh current $\mathbf{I}_1$:

$$\begin{aligned} -12 + 2\mathbf{I}_1 + (3 + j6)(\mathbf{I}_1 - \mathbf{I}_2) &= 0, \\ \mathbf{I}_1(5 + j6) + \mathbf{I}_2[-(3 + j6)] &= 12. \end{aligned} \tag{1}$$

For mesh current $\mathbf{I}_2$:

$$\begin{aligned} j6 + (3 + j6)(\mathbf{I}_2 - \mathbf{I}_1) + 4\mathbf{I}_2 &= 0, \\ \mathbf{I}_1 + \mathbf{I}_2[-(3 + j6)] + \mathbf{I}_2(7 + j6) &= -j6.. \end{aligned} \tag{2}$$

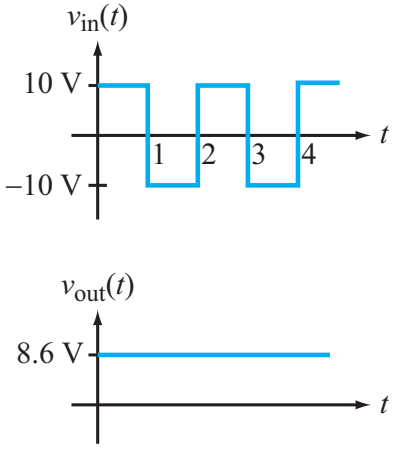
Put (1) and (2) into matrix form:

$$\begin{bmatrix} (5 + j6) & -(3 + j6) \\ -(3 + j6) & (7 + j6) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -j6 \end{bmatrix}.$$

Alternatively, direct application of Eq. (7.108) leads to the same matrix equation.

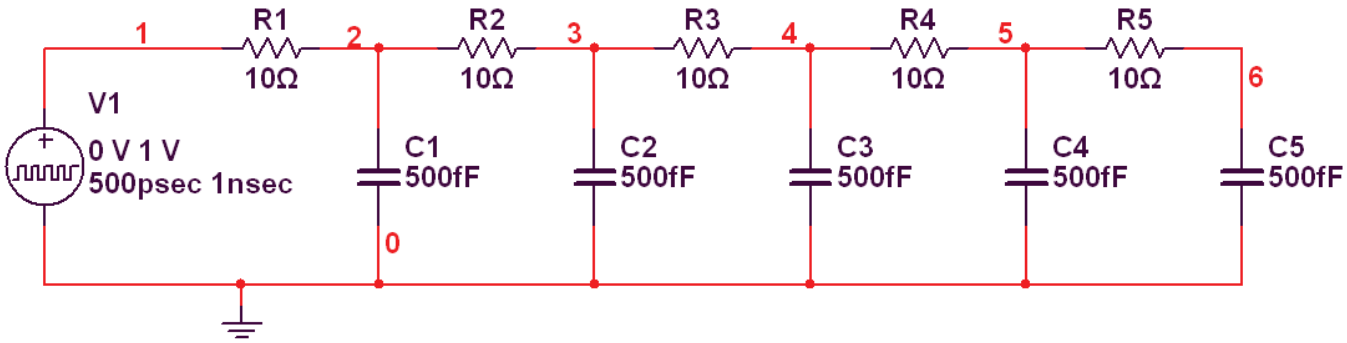
Exercise 7-17 Suppose the input voltage in the circuit of Fig. 7-38 is a 10 V amplitude square wave. What would the output look like?

Solution:

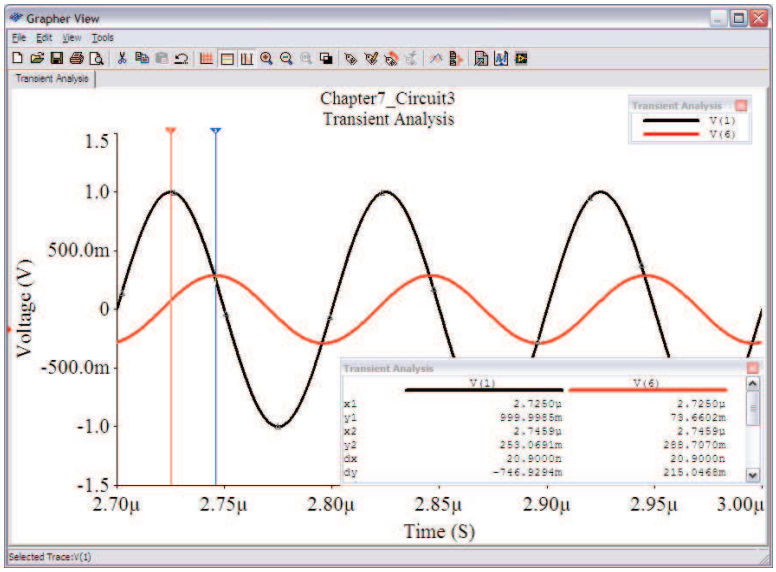


$$\upsilon_{out}(t) = |\upsilon_{in}(t)| - 2V_F$$
$$= 10 - 2 \times 0.7 = 8.6 \text{ V}.$$

Exercise 7-18 Determine the amplitude and phase of V(6) in the circuit of Example 7-21, relative to those of V(1).



Solution:



The figure above shows a plot of V(6) and V(1) as in Fig. 7-42. Note that x1 for V(1) is 2.7250 μs and x2 for V(6) is 2.7459 μs. The time difference between the two values is

$$\Delta t = 2.7459\ \mu\text{s} - 2.7250\ \mu\text{s} = 0.0209\ \mu\text{s},$$

given that $f = 10$ MHz, $T = 0.1\ \mu$s and

$$\phi = 360^\circ \left(\frac{0.0209}{0.1}\right) = 75.24^\circ.$$

Exercise 8-1 Determine the average and rms values of the waveform

$$v(t) = 12 + 6\cos 400t \text{ V}.$$

Solution: From Eq. (8.5),

$$V_{\text{av}} = \frac{1}{T}\int_0^T v(t)\,dt.$$

$$\omega = 2\pi f = \frac{2\pi}{T} = 400 \text{ rad/s},$$

$$T = \frac{2\pi}{400} \text{ s}.$$

$$V_{\text{av}} = \frac{400}{2\pi}\int_0^{2\pi/400} (12 + 6\cos 400t)\,dt$$

$$= \frac{400}{2\pi}\left[\int_0^{2\pi/400} 12\,dt + \int_0^{2\pi/400} 6\cos(400t)\,dt\right].$$

The average of a cosine function over a full cycle is zero. Hence, second term $= 0$.

$$V_{\text{av}} = \frac{400}{2\pi}(12t)\Big|_0^{2\pi/400}$$

$$= 12 \text{ V}.$$

For Eq. (8.14),

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$$

$$= \sqrt{\frac{400}{2\pi}\int_0^{2\pi/400} (12 + 6\cos 400t)^2\,dt}$$

$$= \left[\frac{400}{2\pi}\left(\int_0^{2\pi/400} 144\,dt + \int_0^{2\pi/400} 144\cos 400t\,dt + \int_0^{2\pi/400} 36\cos^2(400t)\,dt\right)\right]^{1/2}.$$

First term $= 144$
Second term $= 0$
Third term:

$$\frac{400}{2\pi}\int_0^{2\pi/400} 36\cos^2(400t)\,dt = 18.$$

Hence,

$$V_{\text{RMS}} = \sqrt{144 + 0 + 18} = 12.73 \text{ V}.$$

Exercise 8-2 Determine the average and rms value of the waveform

$$i(t) = 8\cos 377t - 4\sin(377t - 30^\circ) \text{ A}.$$

Solution: Determine average and rms value of the waveform.

I_{av}**:** Average of sinusoid is zero so the average of the sum of two sinusoids is also zero.

Hence, $I_{\text{av}} = 0$.

I_{rms}**:** From Eq. (8.14):

$$\begin{aligned}
I_{\text{rms}} &= \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt} \\
&= \left[\frac{377}{2\pi}\int_0^{2\pi/377} (8\cos(377t) - 4\sin(377t - 30^\circ))^2\,dt\right]^{1/2} \\
&= \left\{\frac{377}{2\pi}\left[64\cos^2(377t) - 64\cos(377t)\sin(377t - 30^\circ)\right.\right. \\
&\qquad \left.\left. + 16\sin^2(377t - 30^\circ)\right]\,dt\right\}^{1/2} \\
&= \left\{\frac{377}{2\pi}\left[64\int_0^{2\pi/377}\cos^2(377t)\,dt\right.\right. \\
&\qquad - 64\int_0^{2\pi/377}\cos(377t)\sin(377t - 30^\circ)\,dt \\
&\qquad \left.\left. + 16\int_0^{2\pi/377}\sin^2(377t - 30^\circ)\,dt\right]\right\}^{1/2}.
\end{aligned}$$

Use the following identities:

$$\int \cos^2 x\,dx = \frac{1}{4}\sin(2x) + \frac{x}{2}$$

$$\int \sin^2 x\,dx = -\frac{1}{4}\sin(2x) + \frac{x}{2}$$

$$\int \cos(ax)\sin(ax + b)\,dx = \frac{x}{2}\sin(b) = \frac{\cos(2ax + b)}{4a}$$

For first term:

$$\int_0^{2\pi/377} \cos^2(377t)\,dt = \frac{1}{377}\int_0^{2\pi} \cos^2(x)\,dx$$
$$= \frac{1}{377}\left[\frac{1}{4}\sin(2x)+\frac{x}{2}\right]\Bigg|_0^{2\pi} = \frac{\pi}{377}\,.$$

For second term:

$$\int_0^{2\pi/377} \cos(377t)\sin(377t-30^\circ)\,dt = \frac{t}{2}\sin(-30^\circ) - \frac{\cos(754t-30^\circ)}{1508}\Bigg|_0^{2\pi/377}$$
$$= \frac{\pi}{377}\sin(-30^\circ) - \frac{\cos(-30^\circ)}{1508} + \frac{\cos(-30^\circ)}{1508}$$
$$= \frac{\pi}{377}\sin(-30^\circ).$$

For third term:

$$\int_0^{2\pi/377} \sin^2(377t-30^\circ)\,dt = \frac{1}{377}\left[-\frac{1}{4}\sin(-60^\circ)+\frac{1}{4}\sin(-60^\circ)+\frac{2\pi-30^\circ}{2}+\frac{30^\circ}{2}\right]$$
$$= \frac{\pi}{377}\,.$$

$$I_{\text{rms}} = \left\{\frac{377}{2\pi}\left[\frac{64\pi}{377} - \frac{64\pi}{377}\sin(-30^\circ)+\frac{16\pi}{377}\right]\right\}^{1/2}$$
$$= (56)^{1/2}$$
$$= 7.48 \text{ A}.$$

Exercise 8-3 The voltage across and current through a certain load are given by:

$$v(t) = 8\cos(754t - 30^\circ) \text{ V},$$
$$i(t) = 0.2\sin 754t \text{ A}.$$

What is the average power consumed by the load, and by how far in time is $i(t)$ shifted relative to $v(t)$?

Solution: From the expressions for the voltage and current:

$$\phi_v = -30^\circ, \qquad V_m = 8 \text{ V},$$
$$\phi_i = -90^\circ, \qquad I_m = 0.2 \text{ A}.$$

Using Eq. (8.23),

$$\begin{aligned} P_{av} &= \frac{V_m I_m}{2}\cos(\phi_v - \phi_i) \\ &= \frac{8 \times 0.2}{2}\cos(-30^\circ - (-90^\circ)) \\ &= 0.4 \text{ W}. \end{aligned}$$

Also,

$$\phi_v - \phi_i = -30^\circ - (-90^\circ) = 60^\circ = \frac{\pi}{3} \text{ radians}.$$

Hence,

$$\Delta t = \frac{(\phi_v - \phi_i}{\omega} = \frac{\frac{\pi}{3} \text{ (radians)}}{754 \left(\frac{\text{radians}}{\text{second}}\right)} = 1.39 \text{ ms}.$$

Exercise 8-4 The current flowing into a load is given by $i(t) = 2\cos 2500t$ (A). If the load is known to consist of a series of two passive elements, and $\mathbf{S} = (10 - j8)$ (VA), determine the identities of the elements and their values.

Solution: From the expression for $i(t)$:

$$I_m = 2 \text{ A}, \qquad \text{and } \phi_i = 0^\circ.$$

From Eq. (8.31b),

$$\mathbf{I}_{rms} = \frac{I_m}{\sqrt{2}} e^{j\phi_i} = \frac{2}{\sqrt{2}} = 1.414 \text{ A}.$$

From the expression for $\mathbf{S}$,

$$P_{av} = 10 \text{ W}.$$

Using Eq. (8.40a),

$$P_{av} = I_{rms}^2 R \quad \Longrightarrow \quad R = \frac{10}{(1.414)^2} = 5\ \Omega.$$

Also,

$$Q = -8 \text{ (VAR)}.$$

Using Eq. (8.40b),

$$Q = I_{rms}^2 X$$

$$X = \frac{-8}{(1.414)^2} = -4\ \Omega.$$

Hence,

$$-j4 = \frac{-j}{\omega C}.$$

At $\omega = 2500$ rad/s,

$$C = \frac{1}{4\omega} = 100\ \mu\text{F}.$$

Exercise 8-5 At 60 Hz, the impedance of a RL load is $\mathbf{Z}_L = (50 + j50)\ \Omega$. (a) What is the value of the power factor of $\mathbf{Z}_L$ and (b) what will be the new power factor if a capacitance $C = \frac{1}{12\pi}$ mF is added in parallel with the RL load?

Solution:

(a)

$$R = 50\ \Omega, \qquad X = 50\ \Omega,$$

$$\phi_{Z_1} = \tan^{-1}\left(\frac{X}{R}\right)$$

$$= \tan^{-1}\left(\frac{50}{50}\right) = 45^\circ.$$

From Eq. (8.49a),

$$pf_1 = \cos(\phi_Z)$$

$$= \cos(45^\circ)$$

$$= 0.707.$$

(b) With $\left(\frac{1}{12}\ \pi\right)$ mF capacitor at 60 Hz:

$$\mathbf{Z}_C = \frac{-j}{\left(\frac{10^{-3}}{12\pi}\right)(60)(2\pi)}$$

$$= -j100\ \Omega.$$

Parallel combination:

$$\mathbf{Z}_C \parallel \mathbf{Z}_L = \frac{(-j100)(50 + j50)}{50 - j50} = 100\ \Omega.$$

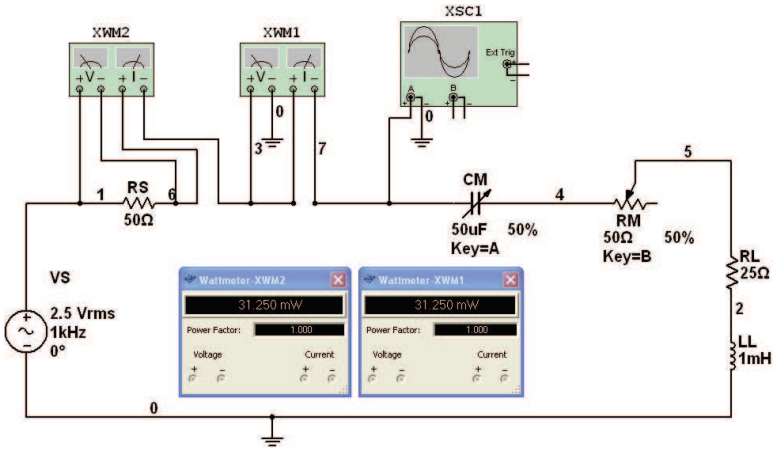
For purely real impedance,

$$\phi_Z = \tan^{-1}(0) = 0^\circ,$$

and

$$pf_2 = \cos(0^\circ) = 1.$$

Exercise 8-6 Use Multisim to simulate the circuit in Fig. 8-15. Connect Channel B of the oscilloscope across the voltage source $\mathbf{V}_s$. Vary C_M over its full range, noting the phase difference between the two channels of the oscilloscope at $C_M = 0$, $C_M = 25\ \mu$F, and $C_M = 50\ \mu$F.



Solution: The best way to observe the changes that C_M has on the phase difference between VS and V7 is to run the Interactive Simulation, but set the Maximum Time Step (under Simulate → Interactive Simulation Settings → Maximum Time Step) to something very small (e.g., 1e-7 seconds). This will force the oscilloscope trace to plot slowly with respect to any interactive adjustment of the value of C_M (using the mouse slider or the keyboard).

- Notice there is a small phase difference at $C_M = 50\ \mu$F. Fig. E8-8a shows the plot of the oscilloscope trace with cursors.

  For $C_M = 50\ \mu$F, x1 on VS $= 1.2500$ ms, x2 on V7 $= 1.2449$ ms.

  $\Delta t = 10.2500\text{ ms} - 10.2449\text{ ms} = 0.0051$ ms.

  Given that $f = 1$ kHz, $T = 1$ ms and

$$\phi = 360^\circ \left(\frac{0.0051}{1}\right) = 1.836^\circ.$$

- As one reduces the value of C_M, the phase difference disappears. For $C_M = 25\ \mu$F, $\Delta t = 0$ ms.
- However, below 25 μF, the phase difference increases to a maximum difference of -19.47°.
- For $C_M = 0\ \mu$F, $\Delta t = 0$ ms.

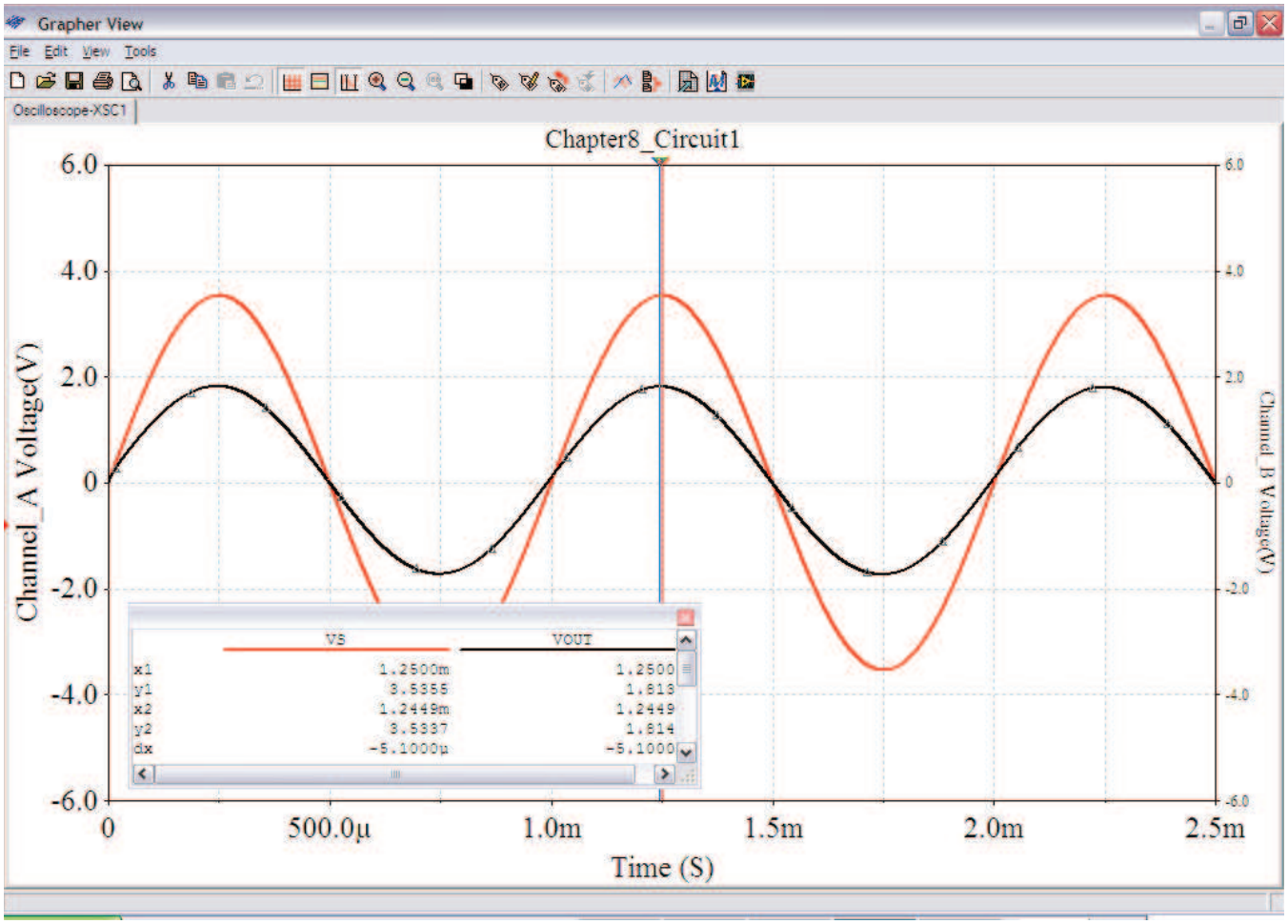


Figure E8.6a

Figure E8-6b plots the phase difference as a function of C_M. This plot was generated in MATLAB software using Eq. (8.78) and voltage division:

$$\angle N_7 = \angle \left[\left(\frac{R_s}{R_s + Z_{\text{Load+Match}}} \right) V_s \right].$$

Note: The *angle* function in MATLAB software can be used to plot the phase angle in radians of any function.

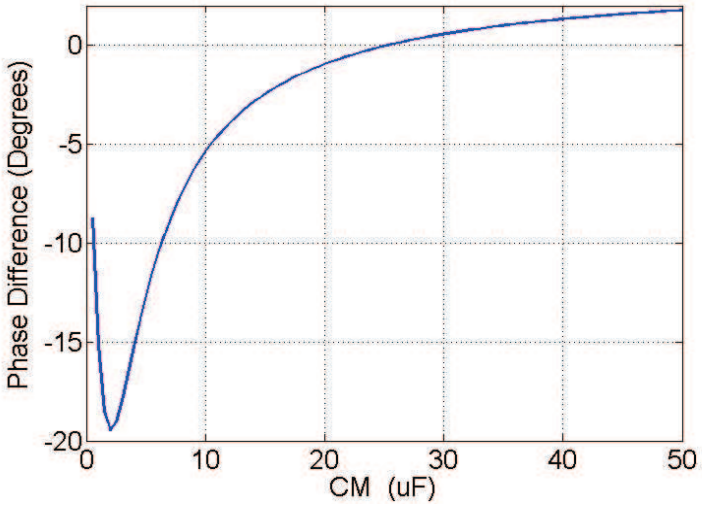
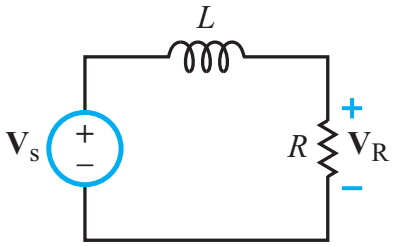


Figure E8.6b

Exercise 9-1 A series RL circuit is connected to a voltage source $\mathbf{V}_s$. Obtain an expression for $\mathbf{H}(\omega) = \mathbf{V}_R/\mathbf{V}_s$, where $\mathbf{V}_R$ is the phasor voltage across R. Also, determine the corner frequency of $\mathbf{H}(\omega)$.

Solution:



By voltage division,

$$\mathbf{V}_R = \frac{R}{R + \mathbf{Z}_L}\,\mathbf{V}_s,$$

or

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_R}{\mathbf{V}_s} = \frac{R}{R + j\omega L} = \frac{R(R - j\omega L)}{R^2 + \omega^2 L^2}\,.$$

Corner frequency ω_c is the value of ω at which the magnitude of $\mathbf{H}(\omega)$ is equal to $1/\sqrt{2}$ of its peak value. The magnitude of $\mathbf{H}(\omega)$ is

$$\mathbf{M}(\omega) = |\mathbf{H}(\omega)| = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}\,,$$

and its peak value is at $\omega = 0$. Thus

$$M_0 = M(\omega)|_{\max} = 1.$$

Setting $\mathbf{H}(\omega_c) = 1/\sqrt{2}$ is equivalent to

$$\frac{1}{2} = \frac{R^2}{R^2 + \omega_c^2 L^2}\,,$$

whose solution is

$$\omega_c = \frac{R}{L}\,.$$

Exercise 9-2 Obtain an expression for the input impedance of the circuit in Fig. E9-2, and then use it to determine the resonant frequency.

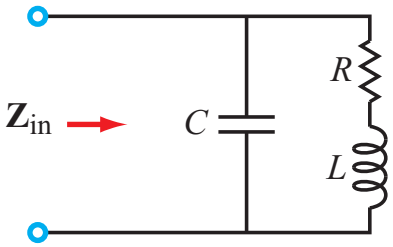


Figure E9.2

Solution:

$$\begin{aligned}
\mathbf{Z}_{\text{in}} &= \left(\frac{1}{\mathbf{Z}_{\text{C}}} + \frac{1}{R+\mathbf{Z}_{\text{L}}}\right)^{-1} \\
&= \frac{\mathbf{Z}_{\text{C}}(R+\mathbf{Z}_{\text{L}})}{R+\mathbf{Z}_{\text{L}}+\mathbf{Z}_{\text{C}}} \\
&= \frac{(1/j\omega C)(R+j\omega L)}{R+j\omega L+(1/j\omega C)} \\
&= \frac{R+j\omega L}{(1-\omega^2 LC)+j\omega RC} \\
&= \frac{[R(1-\omega^2 LC)+\omega^2 RLC]+j[\omega L(1-\omega^2 LC)-\omega R^2 C]}{(1-\omega^2 LC)^2+\omega^2 R^2 C^2}\,.
\end{aligned}$$

Resonance occurs when imaginary part of $\mathbf{Z}_{\text{in}}$ is zero. Thus,

$$\omega_0 L(1-\omega_0^2 LC)-\omega_0 R^2 C = 0,$$

which leads to

$$\omega_0 = 0,$$

which is a trivial solution, and

$$\omega_0 = \sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}\,.$$

Exercise 9-3 Determine: (a) $\mathbf{Z}_{\text{in}}$ of the prototype circuit shown in Fig. E9-3 at $\omega = 1$ rad/s and (b) $\mathbf{Z}'_{\text{in}}$ of the same circuit after scaling it by $K_{\text{m}} = 1000$ and $K_{\text{f}} = 1000$.

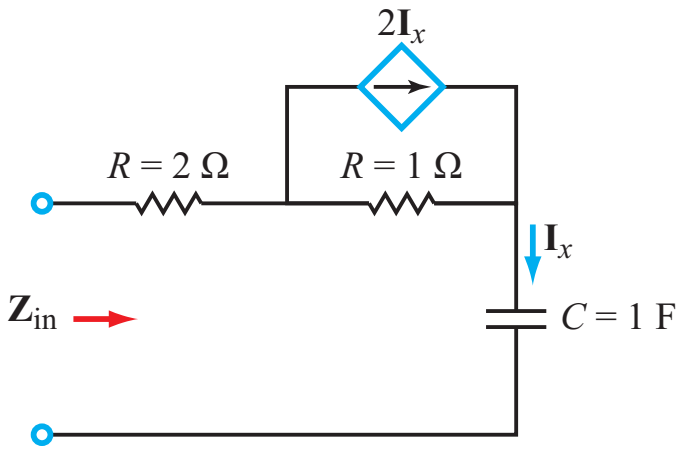
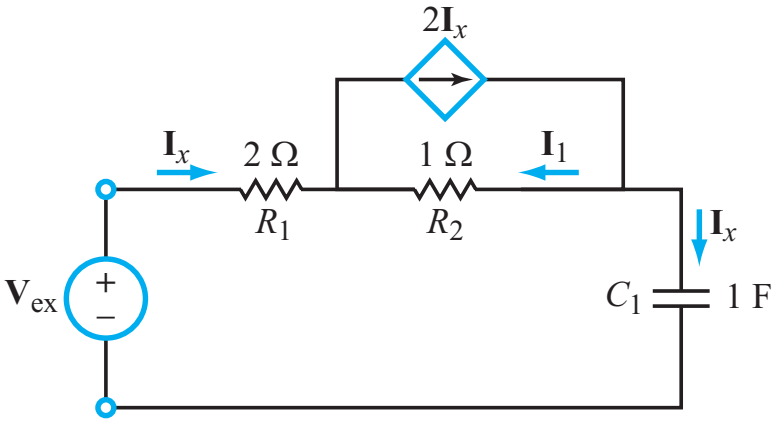


Figure E9.3

Solution:



(a) Since the circuit contains a dependent source, we use the external voltage source method. Also, current continuity requires that the current flowing through the 2-Ω resistor be the same as that flowing through the capacitor. Also $\mathbf{I}_1 = \mathbf{I}_x$.

KVL gives

$$-\mathbf{V}_{\text{ex}} + 2\mathbf{I}_x - \mathbf{I}_x + \mathbf{I}_x\mathbf{Z}_{\text{C}} = 0,$$

which leads to

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_{\text{ex}}}{\mathbf{I}_x} = (1 - j1)\ \Omega \qquad [\text{for } \omega = 1 \text{ rad/s and } C = 1 \text{ F}].$$

(b) With $k_m = 1000$ and $k_f = 1000$,

$$R_1' = 1000R_1 = 1\ \text{k}\Omega,$$
$$R_2' = 1000R_2 = 2\ \text{k}\Omega,$$
$$C' = \frac{1}{k_m k_f} C = 1\ \mu\text{F},$$
$$\omega' = k_f \omega = 10^3\ \text{rad/s}.$$

Repeat of solution with new values leads to

$$\mathbf{Z}_{in}' = (1 - j1)\ \text{k}\Omega.$$

Exercise 9-4 Convert the following voltage ratios to dB: (a) 20, (b) 0.03, (c) 6×10^6.

Solution:
(a) $20\log 20 = 20 \times 1.301 = 26.02$ dB.
(b) $20\log 0.03 = 20 \times (-1.523) = -30.46$ dB.
(c) $20\log(6 \times 10^6) = 20\log 6 + 20\log 10^6 = 15.56 + 120 = 135.56$ dB.

Exercise 9-5 Convert the following dB values to voltage ratios: (a) 36 dB, (b) -24 dB, (c) -0.5 dB.

Solution:
(a) $(10)^{36/20} = 63.1$.
(b) $(10)^{-24/20} = 0.063$.
(c) $(10)^{-0.5/20} = 0.094$.

Exercise 9-6 Generate a Bode magnitude plot for the transfer function

$$\mathbf{H} = \frac{10(100 + j\omega)(1000 + j\omega)}{(10 + j\omega)(10^4 + j\omega)}.$$

Solution: We start by converting the transfer function into standard form:

$$\begin{aligned}\mathbf{H} &= \frac{10 \times 100(1 + j\omega/100) \times 1000 \times (1 + j\omega/1000)}{10(1 + j\omega/10) \times 10^4(1 + j\omega/10^4)} \\ &= \frac{10(1 + j\omega/\omega_{c_2})(1 + j\omega/\omega_{c_3})}{(1 + j\omega/\omega_{c_1})(1 + j\omega/\omega_{c_4})},\end{aligned}$$

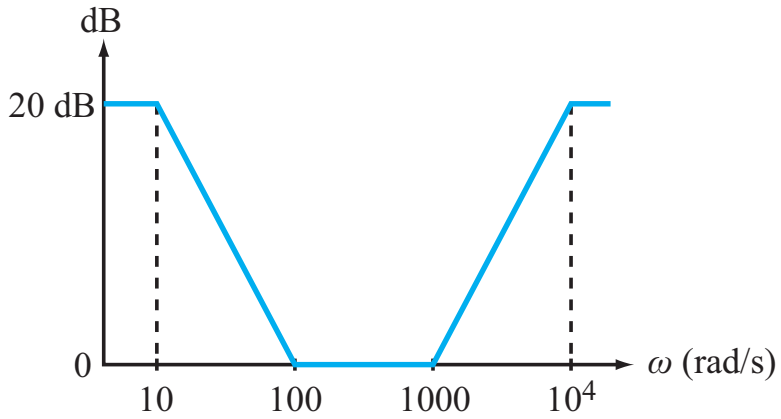
where

$$\begin{aligned}\omega_{c_1} &= 10 \text{ rad/s}, \\ \omega_{c_2} &= 100 \text{ rad/s}, \\ \omega_{c_3} &= 10^3 \text{ rad/s}, \\ \omega_{c_4} &= 10^4 \text{ rad/s}.\end{aligned}$$

We note:

constant term 10 $\Longrightarrow$ 20 dB
zero factors with ω_{c_2} and ω_{c_3}
pole factors with ω_{c_1} and ω_{c_4}.

Sequential addition of terms leads to the solution shown.



Exercise 9-7 Determine the functional form of the transfer function whose Bode magnitude plot is shown in Fig. E9-7, given that its phase angle at dc is 90°.

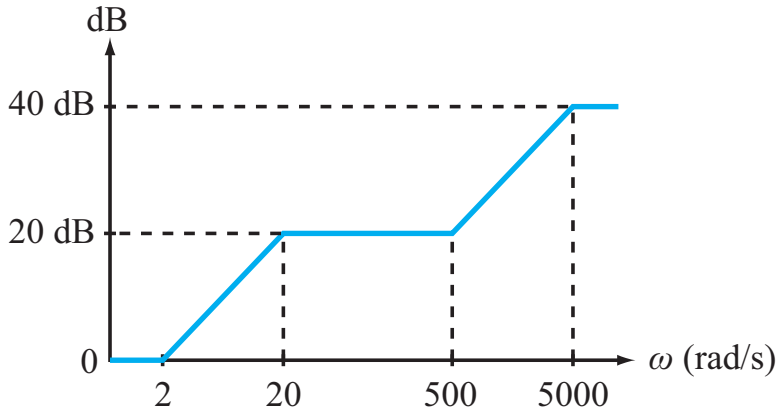


Figure E9.7

Solution: From the plot, the transfer function has:

(1) simple zero factor at $\omega_{c_1} = 2$ rad/s,

(2) simple pole factor at $\omega_{c_2} = 20$ rad/s,

(3) simple zero factor at $\omega_{c_3} = 500$ rad/s,

(4) simple pole factor at $\omega_{c_4} = 5000$ rad/s.

Since at dc, the phase is 90°, we add a j to the numerator.

$$\mathbf{H}(\omega) = \frac{j(1 + j\omega/2)(1 + j\omega/500)}{(1 + j\omega/20)(1 + j\omega/5000)}.$$

Exercise 9-8 Show that for the parallel RLC circuit shown in Fig. E9-8, the transfer-impedance transfer function $\mathbf{H_Z} = \mathbf{V}_R/\mathbf{I}_s$ exhibits a bandpass-filter response.

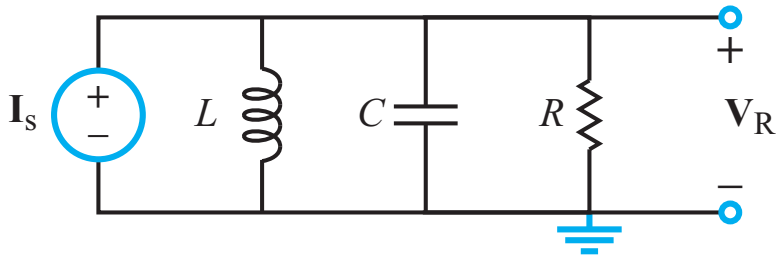


Figure E9.8

Solution: From KCL,

$$\mathbf{V}_R\left(\frac{1}{R}+\frac{1}{\mathbf{Z}_C}+\frac{1}{\mathbf{Z}_L}\right)=\mathbf{I}_s,$$

we obtain the expression

$$\mathbf{H_Z}=\frac{\mathbf{V}_R}{\mathbf{I}_s}=\frac{j\omega L}{(1-\omega^2LC)+j\omega L/R}$$
$$=\frac{\omega^2L^2-j\omega LR^2(1-\omega^2LC)}{R^2(1-\omega^2LC)^2+\omega^2L^2}.$$

Resonance occurs when imaginary component is zero:

$$\omega_0=0 \qquad \text{(trivial)},$$
$$\omega_0=\frac{1}{\sqrt{LC}}.$$

Exercise 9-9 How should R be related to L and C so that the denominator of Eq. (9.66) becomes a simple pole of order 2? What will the value of Q be in that case?

Solution: Denominator of Eq. (9.66):

$$(1-\omega^2 LC)+j\omega RC = 1+(j\omega\sqrt{LC})^2+j\omega RC.$$

For perfect square $(1+2x+x^2)$,

$$j\omega RC = 2(j\omega\sqrt{LC}),$$

or

$$R^2C^2 = 4LC,$$

which gives

$$R = 2\sqrt{\frac{L}{C}}\,.$$

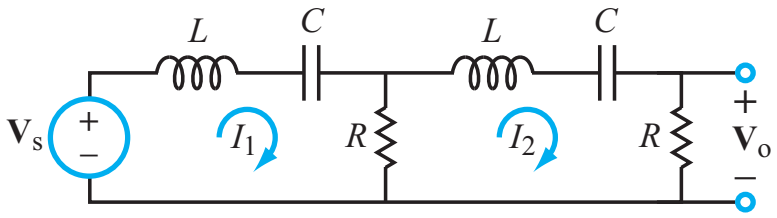
From Eq. (9.61),

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\sqrt{LC}} \times \frac{L}{2\sqrt{L/C}} = \frac{1}{2}\,.$$

Exercise 9-10 Is $M_{BR} = 1 - M_{BP}$?

Solution: No, because $M_{BR} = |\mathbf{H}_{BR}| = |1 - \mathbf{H}_{BP}| \neq 1 - |\mathbf{H}_{BP}| = 1 - M_{BP}$.

Exercise 9-11 What is the order of the two-stage bandpass-filter circuit shown in Fig. 9-18(a)?



Solution: Intuitively, the circuit suggests that each stage is a single order, so the two stages lead to a second-order filter. To be certain, however, we should analyze the expression for $\mathbf{H}(\omega)$ given in Example 9-8 as

$$\mathbf{H}(\omega) = \frac{\omega^2 R^2 C^2 [\omega^2 R^2 C^2 - (1 - \omega^2 LC)^2 + j3\omega RC(1 - \omega^2 LC)]}{[\omega^2 R^2 C^2 - (1 - \omega^2 LC)^2]^2 + 9\omega^2 R^2 C^2 (1 - \omega^2 LC)^2}$$

When the imaginary part is zero, the resonant frequency for $L = 10$ mH, $C = 1\ \mu$F and $R = 2\ \Omega$ is

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^4 \text{ rad/s},$$

and

$$RC = 2 \times 10^{-6} \text{ s}.$$

1. For $\omega \ll \omega_0$, $\omega^2 LC \ll 1$ and $\omega RC \ll 1$. Hence, $\mathbf{H}(\omega)$ simplifies to

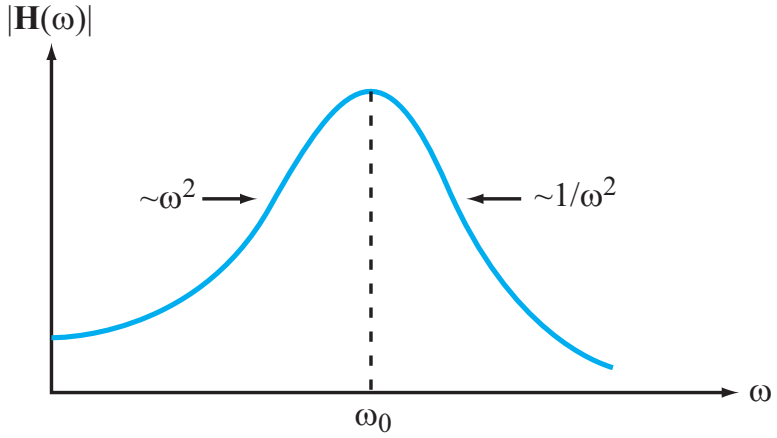
$$\mathbf{H}(\omega) \simeq \omega^2 R^2 C^2 \qquad \text{for } \omega \ll \omega_0.$$

Thus, the power of ω is 2, suggesting that the filter is second order.

2. For $\omega \gg \omega_0$, $\omega^2 LC \gg 1$, and

$$\mathbf{H}(\omega) \simeq \frac{\omega^6 R^2 L^2 C^4}{\omega^8 L^4 C^4} = \frac{R^2}{\omega^2 L^2} \qquad \text{for } \omega \gg \omega_0.$$

The power of ω is -2, again confirming that $\mathbf{H}(\omega)$ is a second-order filter.



Exercise 9-12 Determine the order of $\mathbf{H}(\omega) = \mathbf{V}_{out}/\mathbf{V}_s$ for the circuit in Fig. E9-12.

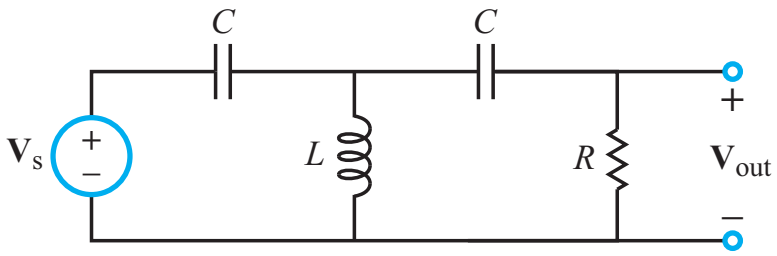


Figure E9.12

Solution: Circuit analysis leads to

$$\mathbf{H}(\omega) = \frac{j\omega^3 RLC^2}{\omega^2 LC - (1 - \omega^2 LC)(1 + j\omega RC)}.$$

For ω very large, such that $\omega^2 LC >> 1$ and $\omega RC \gg 1$,

$$\mathbf{H}(\omega) \simeq 1, \qquad \omega \text{ very large.}$$

For ω very small, such that $\omega^2 LC \ll 1$ and $\omega RC \ll 1$,

$$\mathbf{H}(\omega) \simeq j\omega^3 RLC^2.$$

Hence, filter is third order.

Exercise 9-13 Choose values for R_s and R_f in the circuit of Fig. 9-23(b) so that the gain magnitude is 10 and the corner frequency is 10^3 rad/s, given that $C_f = 1\ \mu$F.

Solution: According to Eq. (8.89),

$$G_{LP} = -\frac{R_f}{R_s} = -10,$$

$$\omega_{LP} = \frac{1}{R_f C_f} = 10^3 \text{ rad/s}.$$

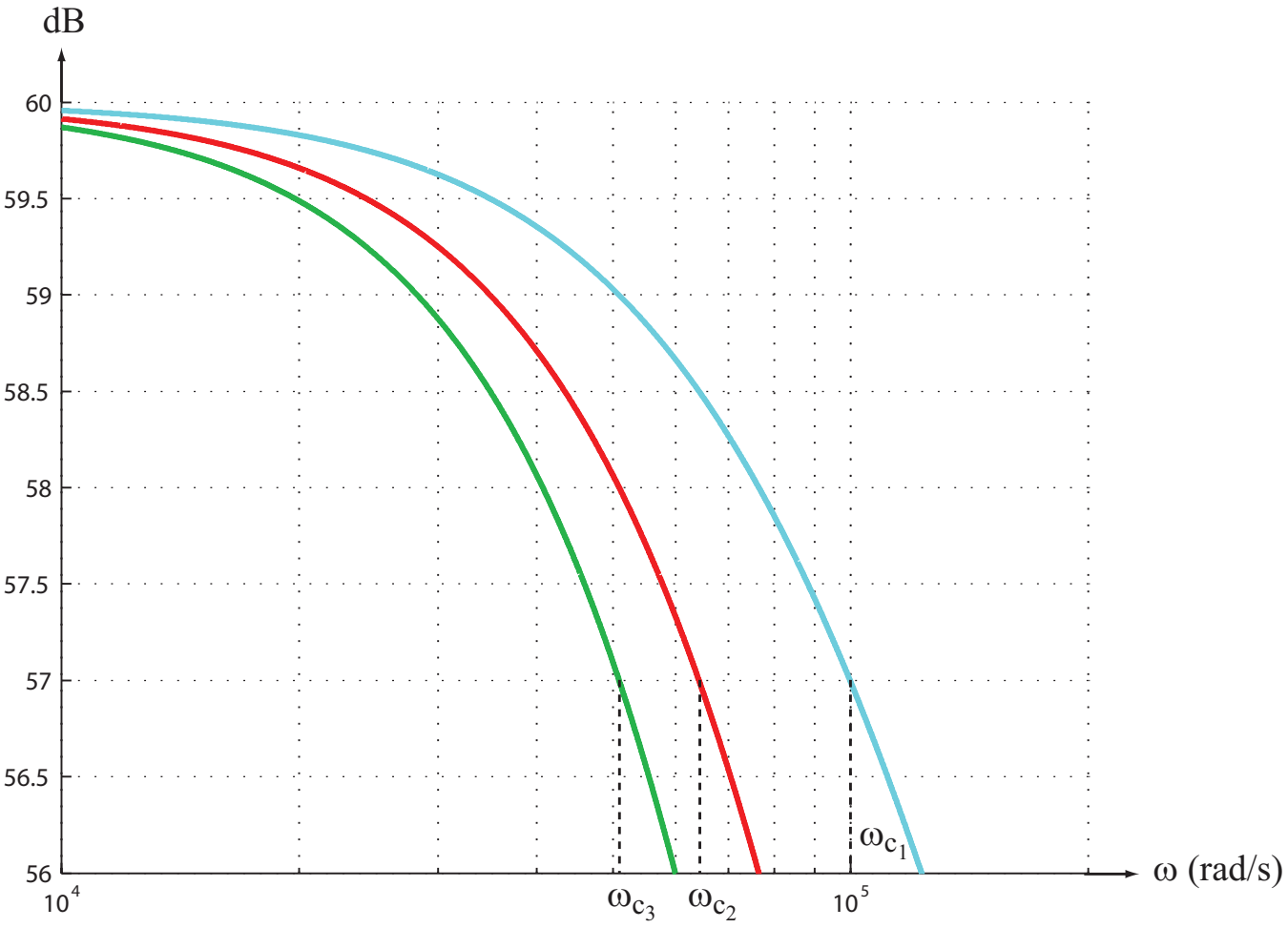
With $C_f = 1\ \mu$F,

$$R_f = 1\ \text{k}\Omega, \qquad \text{and } R_s = 100\ \Omega.$$

Exercise 9-14 What are the values of the corner frequencies associated with M_1, M_2, and M_3 of Example 9-10?

Solution: By plotting the expressions for M_1, M_2, and M_3 and determining the angular frequencies at which each is $1/\sqrt{2}$ of its peak value, we can show that

$$\omega_{c_1} = 10^5 \text{ rad/s}, \quad \omega_{c_2} = 0.64\omega_{c_1}, \quad \text{and } \omega_{c_3} = 0.51\omega_{c_1}.$$



Exercise 9-15 The bandreject filter of Example 9-12 uses two lowpass filter stages and two highpass filter stages. If three stages of each were used instead, what would the expression for $\mathbf{H}(\omega)$ be in that case?

Solution:

$$\mathbf{H}(\omega) = 50\left[\left(\frac{1}{1+j\omega/4\pi\times 10^4}\right)^3 + \left(\frac{j\omega/8\pi\times 10^4}{1+j\omega/8\pi\times 10^4}\right)^3\right].$$

Exercise 10-1 Superimpose onto Fig. 10-4(b) the three source voltages of the Δ configuration.

Solution:

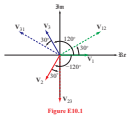


Figure E10.1

Exercise 10-2 Given a balanced Δ-source configuration with a positive phase sequence and $\mathbf{V}_{12} = 208\angle 45^\circ$ V (rms), determine (a) phase voltages $\mathbf{V}_{23}$ and $\mathbf{V}_{31}$, and (b) $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ of the equivalent Y-source configuration.

Solution: (a) For a positive phase sequence, $\mathbf{V}_{23}$ and $\mathbf{V}_{31}$ have the same amplitude as $\mathbf{V}_{12}$, but their phases are retarded by 120° and 240°, respectively. Hence,

$$\mathbf{V}_{23} = 208\angle 45^\circ - 120^\circ = 208\angle -75^\circ \text{ V (rms)},$$
$$\mathbf{V}_{31} = 208\angle 45^\circ - 240^\circ = 208\angle -195^\circ \text{ V (rms)}.$$

(b)

$$\mathbf{V}_1 = \frac{\mathbf{V}_{12}}{\sqrt{3}} \angle -30^\circ$$
$$= \frac{208}{\sqrt{3}} \angle 45^\circ - 30^\circ = 120\angle 15^\circ \text{ V (rms)}$$
$$\mathbf{V}_2 = \mathbf{V}_1 \angle -120^\circ = 120\angle -105^\circ \text{ V (rms)}$$
$$\mathbf{V}_3 = \mathbf{V}_1 \angle -240^\circ = 120\angle -225^\circ \text{ V (rms)}.$$

Exercise 10-3 Show graphically why the phase magnitude of $\mathbf{V}_{12}$ of the Δ-source is $\sqrt{3}$ times larger than the phase magnitude of the Y-source.

Solution:

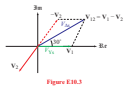
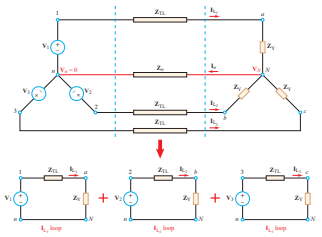


Figure E10.3

Exercise 10-4 Were we to repeat Example 10-2, but with the transmission-line impedances set to zero, which of the following line-current quantities will change and which will remain the same: (a) amplitudes, (b) absolute phases, and (c) phases relative to each other?

Solution: (a) Amplitudes will change, (b) absolute phases will change, but (c) relative phases will continue to be 120° apart (between pairs).

Exercise 10-5 Determine $\mathbf{I}_{L_1}$ in the balanced Y-Y network of Fig. 10-8, given that $\mathbf{V}_1 = 120\angle 0^\circ$ V, $\mathbf{Z}_{TL} = (2+j1)\ \Omega$ and $\mathbf{Z}_Y = (28+j9)\ \Omega$.



Solution:

$$\mathbf{I}_{L_1} = \frac{\mathbf{V}_1}{\mathbf{Z}_{TL}+\mathbf{Z}_Y} = \frac{120}{(2+j1)+(28+j9)}$$

$$= \frac{120}{30+j10}$$

$$= \frac{120}{31.6e^{j18.4^\circ}} = 3.80\angle -18.4^\circ \text{ A}.$$

Exercise 10-6 Prove Eq. (10.26b).

Solution:

$$\begin{aligned}
&\cos\theta + \cos(\theta - 120^\circ) + \cos(\theta - 240^\circ) \\
&\quad = \cos\theta + \cos\theta\cos 120^\circ + \sin\theta\sin 120^\circ + \cos\theta\cos 240^\circ + \sin\theta\sin 240^\circ \\
&\quad = \cos\theta - 0.5\cos\theta + 0.86\sin\theta - 0.5\cos\theta - 0.86\sin\theta \\
&\quad = 0.
\end{aligned}$$

Exercise 10-7 Suppose the circuit shown in Fig. 10-15(a) contains only balanced loads 1 and 2. What value should C have in order to raise the source's power factor to 0.92 lagging?

Solution: With only loads 1 and 2,

$$\begin{aligned}\mathbf{S}_{\mathrm{T}} &= \mathbf{S}_1 + \mathbf{S}_2 \\ &= (4800 + j3600) + (7200 + j9600) \\ &= 12000 + j13200 \text{ VA} \\ \phi &= \tan^{-1}\left(\frac{13200}{12000}\right) = 47.73^\circ \\ pf_{\mathrm{s}} &= \cos\phi = 0.67.\end{aligned}$$

To raise it to $pf'_{\mathrm{s}} = 0.92$, ϕ has to change to

$$\begin{aligned}\phi' &= \cos^{-1}(0.92) = 23.07^\circ \\ \mathbf{S}'_{\mathrm{T}} &= 12000 + j12000\tan\phi' \\ &= (12000 + j5111) \text{ VA} \\ Q_{\mathrm{C}} &= 5111 - 13200 = -8089 \text{ VAR}.\end{aligned}$$

Also

$$Q_{\mathrm{C}} = -2V_{\mathrm{L}}^2\omega C,$$

which leads to

$$C = \frac{-Q_{\mathrm{C}}}{2V_{\mathrm{L}}^2\omega} = \frac{8089}{2\times(1200)^2\times 2\pi\times 60} = 7.45\ \mu\text{F}.$$

Exercise 10-8 When used on a balanced three-phase load, the two-wattmeter method provided measurements $P_1 = 4{,}800$ W and $P_2 = 10{,}200$ W. What is the total complex power $\mathbf{S}_T$ of the load?

Solution:

$$P_T = P_1 + P_2 = 4800 + 10200 = 15000 \text{ W}$$

$$Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}(10200 - 4800) = 9353 \text{ VAR}.$$

Hence,

$$\mathbf{S}_T = (15000 + j9353) \text{ VA}.$$

Exercise 11-1 Repeat Example 11-1 after moving the dot location on the side of L_2 from the top end of the coil to the bottom.

Solution: Replacing M with $-M$ in Eqs. (11.10a and b) leads to

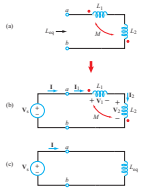
$$-\mathbf{V}_s + \left(R_1 - \frac{j}{\omega C} + j\omega L_1\right)\mathbf{I}_1 + j\omega M\mathbf{I}_2 = 0$$

$$j\omega M\mathbf{I}_1 + \left(j\omega L_2 - \frac{j}{\omega C} + R_L\right)\mathbf{I}_2 = 0.$$

Solution is identical to that in Example 11-1 except that M should be replced with $-M$ in Eq. (11.11). Hence,

$$\begin{aligned} i_L(t) &= 139.5\cos(2\pi \times 10^3 t + 142.2^\circ - 180^\circ) \\ &= 139.5\cos(2\pi \times 10^3 t - 37.8^\circ)\ \text{mA}. \end{aligned}$$

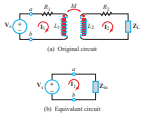
Exercise 11-2 Repeat Example 11-3 for the two in-series inductors in Fig. 11-6(a), but with the dot location on L_2 being on the top end.



Solution: Replacing M with $-M$ everywhere in the expressions for $\mathbf{V}_1$ and $\mathbf{V}_2$ in Example 11-3 leads to

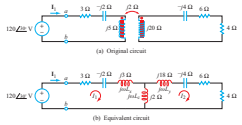
$$\begin{aligned} L_{\text{eq}} &= L_1 + L_2 - (-2M) \\ &= L_1 + L_2 + 2M. \end{aligned}$$

Exercise 11-3 The expression for $\mathbf{Z}_{\text{in}}$ given by Eq. (11.25) was derived for the circuit in Fig. 11-8, in which both dots are on the upper end of the coils. What would the expression look like were the two dots located on opposite ends?



Solution: If the two dots are on opposite ends, M should be replaced with $-M$, but since Eq. (11.24) is proportional to M^2, the expression for $\mathbf{Z}_{\text{in}}$ remains unchanged.

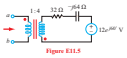
Exercise 11-4 What are the element values of the Π-equivalent circuit of the transformer in Fig. 11-11(a)?



Solution: According to Eqs. (11.31a to c),

$$L_a = \frac{L_1L_2 - M^2}{L_1 - M}$$
$$= \frac{(5/\omega)(20/\omega) - (2/\omega)^2}{(5/\omega) - (2/\omega)} = \frac{32}{\omega},$$
$$L_b = \frac{L_1L_2 - M^2}{L_2 - M} = \frac{5.33}{\omega},$$
$$L_c = \frac{L_1L_2 - M^2}{M} = \frac{48}{\omega}.$$

Exercise 11-5 Determine the Thévenin equivalent of the circuit to the right of terminals (a, b) in Fig. E11-5.



Solution: Application of Fig. 11-15(b) leads to

$$\mathbf{Z}_{\text{Th}} = \frac{32 - j64}{n^2} = \frac{32 - j64}{16} = (2 - j4)\ \Omega,$$

$$\mathbf{V}_{\text{Th}} = -\frac{\mathbf{V}_{s2}}{n} = \frac{-12e^{j60^\circ}}{4} = -3e^{j60^\circ}\ \text{V}.$$

The negative sign accounts for the fact that the dots are on opposite sides of the transformer.

Exercise 11-6 An autotransformer is used to step up the voltage by a factor of 10. If $N = 200$, what are the values of N_1 and N_2?

Solution:

$$\frac{V_2}{V_1} = \frac{N}{N_2},$$

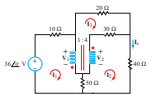
or

$$10 = \frac{200}{N_2} \quad \rightarrow \quad N_2 = 20$$

and

$$N = N_1 + N_2 \quad \rightarrow \quad N_1 = 180.$$

Exercise 11-7 Determine $\mathbf{I}_x$ in the circuit of Fig. 11-18 after replacing the 20-Ω resistor with an open circuit.



Solution: Repeating the solution in Example 11-10 after replacing the 20-Ω resistor with an open circuit gives

$$-36+(10+50)\mathbf{I}_1-50\mathbf{I}_2+\mathbf{V}_1=0,$$
$$-50\mathbf{I}_1+(30+40+50)\mathbf{I}_2-\mathbf{V}_2=0.$$

Also,

$$\mathbf{V}_2=-4\mathbf{V}_1,$$
$$\mathbf{I}_2=-\frac{\mathbf{I}_1}{4}.$$

Solution leads to

$$\mathbf{I}_x=\mathbf{I}_2=-0.097\text{ A}.$$

Exercise 12-1 If $x(t)$ is the rectangular pulse shown in Fig. E12-1(a), determine its time derivative $x'(t)$ and plot it.

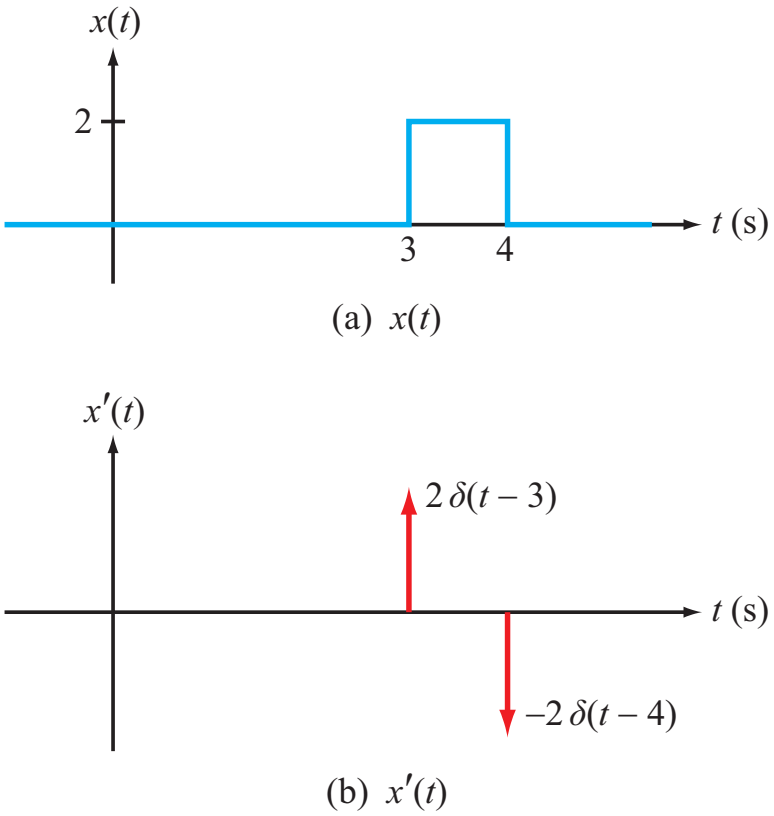


Figure E12.1

Solution:

$$x(t) = 2u(t-3) - 2u(t-4),$$
$$x'(t) = 2\delta(t-3) - 2\delta(t-4).$$

Exercise 12-2 Determine the Laplace transform of (a) $[\sin(\omega t)]\, u(t)$, (b) $e^{-at}\, u(t)$, and (c) $t\, u(t)$. Assume all waveforms are zero for $t < 0$.

Solution:

(a) $[\sin \omega t]\, u(t)$

$$\mathbf{F}(\mathbf{s}) = \int_{0^-}^{\infty} [\sin \omega t]\, u(t)\, e^{-\mathbf{s}t}\, dt.$$

Application of the identity

$$\sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j},$$

$$\begin{aligned}
\mathbf{F}(\mathbf{s}) &= \frac{1}{2j}\int_0^{\infty} e^{j\omega t} e^{-\mathbf{s}t}\, dt - \frac{1}{2j}\int_0^{\infty} e^{-j\omega t} e^{-\mathbf{s}t}\, dt \\
&= \frac{1}{2j}\left(\frac{e^{(j\omega-\mathbf{s})t}}{j\omega-\mathbf{s}} - \frac{e^{-(j\omega+\mathbf{s})t}}{-(j\omega+\mathbf{s})}\right)\Bigg|_0^{\infty} \\
&= \frac{1}{2j}\left(\frac{-1}{j\omega-\mathbf{s}} + \frac{-1}{j\omega+\mathbf{s}}\right) = \frac{\omega}{s^2+\omega^2}.
\end{aligned}$$

(b) $e^{-at}\, u(t)$

$$\begin{aligned}
\mathbf{F}(\mathbf{s}) &= \int_{0^-}^{\infty} e^{-at}\, u(t)\, e^{-\mathbf{s}t}\, dt \\
&= \int_0^{\infty} e^{-(a+\mathbf{s})t}\, dt = \frac{e^{-(a+\mathbf{s})t}}{-(a+\mathbf{s})}\Bigg|_0^{\infty} = \frac{1}{\mathbf{s}+a}.
\end{aligned}$$

(c) $t\, u(t)$

$$\mathbf{F}(\mathbf{s}) = \int_0^{\infty} te^{-\mathbf{s}t}\, dt.$$

Using the integral relation

$$\int xe^{ax}\, dx = \frac{e^{ax}}{a^2}(ax-1),$$

we have

$$\mathbf{F}(\mathbf{s}) = \frac{e^{-\mathbf{s}t}}{\mathbf{s}^2}(-\mathbf{s}t-1)\Bigg|_0^{\infty} = \frac{1}{\mathbf{s}^2}.$$

Exercise 12-3 Determine $\mathscr{L}[\sin \omega(t-T)\, u(t-T)]$.

Solution: According to Exercise 6-9(a),

$$\sin \omega t \quad \longleftrightarrow \quad \frac{\omega}{\mathbf{s}^2+\omega^2} \,.\, \sin \omega t \frac{\omega}{\mathbf{s}^2+\omega^2} \,.$$

Application of the shift property given by Eq. (6.53)

$$f(t-T)\, u(t-T) \quad \longleftrightarrow \quad e^{-T\mathbf{s}}\, F(\mathbf{s})$$

leads to

$$\sin \omega(t-T)\, u(t-T) \quad \longleftrightarrow \quad e^{-T\mathbf{s}}\, \frac{\omega}{\mathbf{s}^2+\omega^2} \,.$$

Exercise 12-4 (a) Prove Eq. (12.27) and (b) apply it to determine $\mathscr{L}[e^{-at}\cos\omega t]$.

Solution:

(a) If

$$f(t) \quad \Longleftrightarrow \quad \mathbf{F}(\mathbf{s}),$$

then

$$\begin{aligned}\int_{0^-}^{\infty} e^{-at} f(t)\, e^{-\mathbf{s}t}\, dt &= \int_{0^-}^{\infty} f(t)\, e^{-(\mathbf{s}+a)t}\, dt \\ &= \int_{0^-}^{\infty} f(t)\, e^{-\mathbf{s}'t}\, dt \\ &= \mathbf{F}(\mathbf{s}') \\ &= \mathbf{F}(\mathbf{s}+a),\end{aligned}$$

where we temporarily used the substitution

$$\mathbf{s}' = \mathbf{s} + a.$$

Hence,

$$e^{-at} f(t) \quad \Longleftrightarrow \quad \mathbf{F}(\mathbf{s}+a).$$

(b) Since

$$\cos\omega t \quad \Longleftrightarrow \quad \frac{\mathbf{s}}{\mathbf{s}^2+\omega^2},$$

it follows that

$$e^{-at}\cos\omega t \quad \Longleftrightarrow \quad \frac{(\mathbf{s}+a)}{(\mathbf{s}+a)^2+\omega^2}.$$

Exercise 12-5 Obtain the Laplace transform of: (a) $f_1(t) = 2(2-e^{-t})\,u(t)$ and (b) $f_2(t) = e^{-3t}\cos(2t+30^\circ)\,u(t)$.

Solution:

(a)

$$\begin{aligned} f_1(t) &= 2(2-e^{-t})\,u(t) \\ &= (4-2e^{-t})\,u(t). \end{aligned}$$

By entries 2 and 3 in Table 12-2,

$$\mathbf{F}_1(\mathbf{s}) = \frac{4}{\mathbf{s}} - \frac{2}{\mathbf{s}+1} = \frac{4\mathbf{s}+4-2\mathbf{s}}{\mathbf{s}(\mathbf{s}+1)} = \frac{2\mathbf{s}+4}{\mathbf{s}(\mathbf{s}+1)}\,.$$

(b)

$$\begin{aligned} f_2(t) &= e^{-3t}\cos(2t+30^\circ)\,u(t) \\ &= e^{-3t}\,f_a(t), \end{aligned}$$

with

$$f_a(t) = \cos(2t+20^\circ)\,u(t).$$

Applying entry #12 in Table 12-2 gives

$$\mathbf{F}_a(\mathbf{s}) = \frac{\mathbf{s}\cos 30^\circ - 2\sin 30^\circ}{\mathbf{s}^2+4} = \frac{0.866\mathbf{s}-1}{\mathbf{s}^2+4}\,.$$

Application of Property 5 in Table 12-1 leads to

$$\begin{aligned} \mathbf{F}_2(\mathbf{s}) &= \mathbf{F}_a(\mathbf{s}+3) \\ &= \frac{0.866(\mathbf{s}+3)-1}{(\mathbf{s}+3)^2+4} \\ &= \frac{0.866\mathbf{s}+1.6}{\mathbf{s}^2+6\mathbf{s}+13}\,. \end{aligned}$$

Exercise 12-6 Apply the partial-fraction expansion method to determine $f(t)$, given that its Laplace transform is

$$\mathbf{F}(\mathbf{s}) = \frac{10\mathbf{s}+16}{\mathbf{s}(\mathbf{s}+2)(\mathbf{s}+4)}\,.$$

Solution: By partial-fraction expansion,

$$\mathbf{F}(\mathbf{s}) = \frac{A_1}{\mathbf{s}} + \frac{A_2}{\mathbf{s}+2} + \frac{A_3}{\mathbf{s}+4}\,,$$

with

$$\begin{aligned}
A_1 &= \mathbf{s}\,\mathbf{F}(\mathbf{s})|_{\mathbf{s}=0} \\
&= \left.\frac{10\mathbf{s}+16}{(\mathbf{s}+2)(\mathbf{s}+4)}\right|_{\mathbf{s}=0} = 2, \\
A_2 &= (\mathbf{s}+2)\,\mathbf{F}(\mathbf{s})|_{\mathbf{s}=-2} \\
&= \left.\frac{10\mathbf{s}+16}{\mathbf{s}(\mathbf{s}+4)}\right|_{\mathbf{s}=-2} = \frac{-20+16}{-2(2)} = 1, \\
A_3 &= (\mathbf{s}+4)\,\mathbf{F}(\mathbf{s})|_{\mathbf{s}=-4} \\
&= \left.\frac{10\mathbf{s}+16}{\mathbf{s}(\mathbf{s}+2)}\right|_{\mathbf{s}=-4} = \frac{-40+16}{-4(-2)} = -3.
\end{aligned}$$

Hence,

$$\mathbf{F}(\mathbf{s}) = \frac{2}{\mathbf{s}} + \frac{1}{\mathbf{s}+2} - \frac{3}{\mathbf{s}+4}\,,$$

and

$$f(t) = [2 + e^{-2t} - 3e^{-4t}]\,u(t).$$

Exercise 12-7 Determine the inverse Laplace transform of

$$\mathbf{F}(\mathbf{s}) = \frac{4\mathbf{s}^2 - 15\mathbf{s} - 10}{(\mathbf{s}+2)^3}.$$

Solution:

$$\mathbf{F}(\mathbf{s}) = \frac{4\mathbf{s}^2 - 15\mathbf{s} - 10}{(\mathbf{s}+2)^3}.$$

By partial-fraction expansion,

$$\mathbf{F}(\mathbf{s}) = \frac{B_1}{\mathbf{s}+2} + \frac{B_2}{(\mathbf{s}+2)^2} + \frac{B_3}{(\mathbf{s}+2)^3},$$

with

$$\begin{aligned}
B_3 &= (\mathbf{s}+2)^3\,\mathbf{F}(\mathbf{s})\big|_{\mathbf{s}=-2} \\
&= 4\mathbf{s}^2 - 15\mathbf{s} - 10\big|_{\mathbf{s}=-2} = 16 + 30 - 10 = 36, \\
B_2 &= \frac{d}{d\mathbf{s}}\left[(\mathbf{s}+2)^3\,\mathbf{F}(\mathbf{s})\right]\Big|_{\mathbf{s}=-2} \\
&= \frac{d}{d\mathbf{s}}\left(4\mathbf{s}^2 - 15\mathbf{s} - 10\right)\Big|_{\mathbf{s}=-2} = 8\mathbf{s} - 15|_{\mathbf{s}=-2} = -31, \\
B_1 &= \frac{1}{2}\,\frac{d}{d\mathbf{s}^2}\left(4\mathbf{s}^2 - 15\mathbf{s} - 10\right)\Big|_{\mathbf{s}=-2} = 4.
\end{aligned}$$

Hence,

$$\mathbf{F}(\mathbf{s}) = \frac{4}{\mathbf{s}+2} - \frac{31}{(\mathbf{s}+2)^2} + \frac{36}{(\mathbf{s}+2)^3}.$$

By entries 3, 6, and 7 in Table 12-2,

$$f(t) = (4 - 31t + 18t^2)e^{-2t}\,u(t).$$

Exercise 12-8 Determine the inverse Laplace transform of

$$\mathbf{F}(\mathbf{s}) = \frac{2\mathbf{s}+14}{\mathbf{s}^2+6\mathbf{s}+25}\,.$$

Solution:

$$\begin{aligned}\mathbf{F}(\mathbf{s}) &= \frac{2\mathbf{s}+14}{\mathbf{s}^2+6\mathbf{s}+25} \\ &= \frac{2\mathbf{s}+14}{(\mathbf{s}+3-j4)(\mathbf{s}+3+j4)}\,.\end{aligned}$$

By partial fraction expansion

$$\mathbf{F}(\mathbf{s}) = \frac{\mathbf{B}_1}{\mathbf{s}+3-j4} + \frac{\mathbf{B}_1^*}{\mathbf{s}+3+j4}\,,$$

with

$$\begin{aligned}\mathbf{B}_1 &= (\mathbf{s}+3-j4)\,\mathbf{F}(\mathbf{s})\big|_{\mathbf{s}=-3+j4} \\ &= \left.\frac{(2\mathbf{s}+14)}{(\mathbf{s}+3+j4)}\right|_{\mathbf{s}=-3+j4} \\ &= \frac{-6+j8+14}{j8} = 1-j = \sqrt{2}\,e^{-j45^\circ}.\end{aligned}$$

Hence,

$$\mathbf{F}(\mathbf{s}) = \frac{\sqrt{2}\,e^{-j45^\circ}}{\mathbf{s}+3-j4} + \frac{\sqrt{2}\,e^{j45^\circ}}{\mathbf{s}+3+j4}\,.$$

By entry #15 in Table 12-2,

$$f(t) = [2\sqrt{2}\,e^{-3t}\cos(4t-45^\circ)]\,u(t).$$

Exercise 12-9 Convert the circuit in Fig. E12.9 into the **s**-domain.

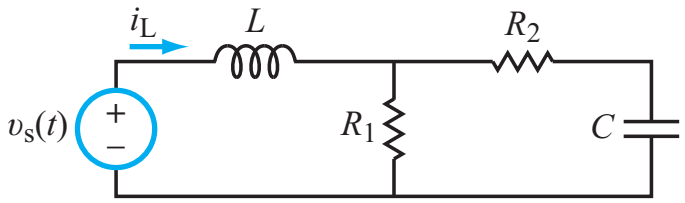
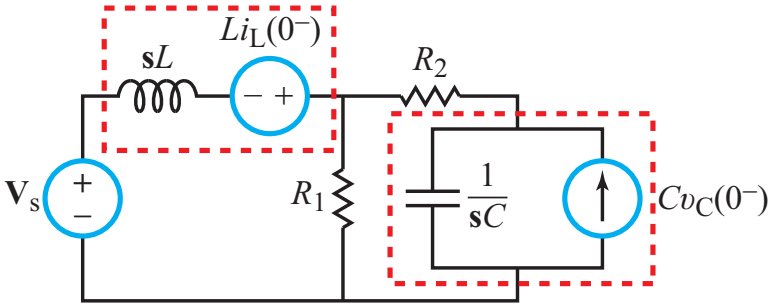


Figure E12.9

Solution:



Exercise 13-1 Obtain the Fourier-series representation for the waveform shown in Fig. E13.1.

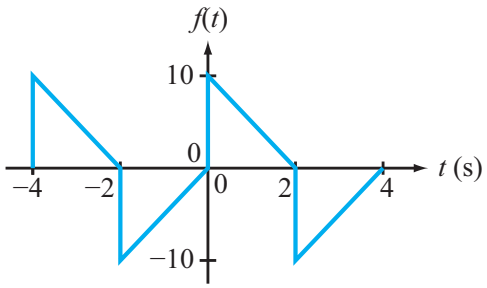


Figure E13.1

Solution: For the cycle from $t = -2$ s to $t = 2$ s, the waveform is given by

$$f(t) = \begin{cases} 5t & \text{for } -2 \le t \le 0 \\ 10 - 5t & \text{for } 0 \le t \le 2. \end{cases}$$

With $T = 4$ s and $\omega_0 = 2\pi/T = \pi/2$ rad/s,

$$\begin{aligned} a_0 &= \frac{1}{T}\int_{-2}^{2} f(t)\,dt \\ &= \frac{1}{4}\left[\int_{-2}^{0} 5t\,dt + \int_{0}^{2}(10-5t)\,dt\right] = 0, \\ a_n &= \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega_0 t\,dt \\ &= \frac{1}{2}\left[\int_{-2}^{0} 5t\cos\frac{n\pi t}{2}\,dt + \int_{0}^{2}(10-5t)\cos\frac{n\pi t}{2}\,dt\right]. \end{aligned}$$

Using the integral relationship given in Appendix D-2 as

$$\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax,$$

we have

$$a_n = \frac{20}{n^2\pi^2}(1 - \cos n\pi).$$

Similarly, using the relation

$$\int x\sin ax\,dx = \frac{1}{a^2}\sin ax + \frac{x}{a}\cos ax,$$

we have

$$\begin{aligned} b_n &= \frac{2}{T}\int_{-T/2}^{T/2} f(t)\ \sin n\omega_0 t\ dt \\ &= \frac{1}{2}\left[\int_{-2}^{0} 5t\sin\frac{n\pi t}{2}\,dt + \int_{0}^{2}(10-5t)\sin\frac{n\pi t}{2}\,dt\right] \\ &= \frac{10}{n\pi}\,(1-\cos n\pi). \end{aligned}$$

Hence,

$$\begin{aligned} f(t) = \sum_{n=1}^{\infty}\Bigg[&\frac{20}{n^2\pi^2}\,(1-\cos n\pi)\cos\frac{n\pi t}{2} \\ &+\frac{10}{n\pi}\,(1-\cos n\pi)\sin\frac{n\pi t}{2}\Bigg]. \end{aligned}$$

Exercise 13-2 Obtain the line spectra associated with the periodic function of Exercise 13.1.

Solution:

$$
\begin{aligned}
A_n &= \sqrt{a_n^2 + b_n^2} \\
&= \left\{ \left[\frac{20}{n^2\pi^2} (1 - \cos n\pi) \right]^2 + \left[\frac{10}{n\pi} (1 - \cos n\pi) \right]^2 \right\}^{1/2} \\
&= (1 - \cos n\pi) \frac{20}{n^2\pi^2} \sqrt{1 + \frac{n^2\pi^2}{4}}, \\
\phi_n &= -\tan^{-1}\left(\frac{b_n}{a_n}\right) \\
&= -\tan^{-1}\left(\frac{n\pi}{2}\right).
\end{aligned}
$$

We note that $A_n = 0$ when $n =$ even.

Exercise 13-3 (a) Does the waveform $f(t)$ shown in Fig. E13.3 exhibit either even or odd symmetry? (b) What is the value of a_0? (c) Does the function $g(t) = f(t) - a_0$ exhibit either even or odd symmetry?

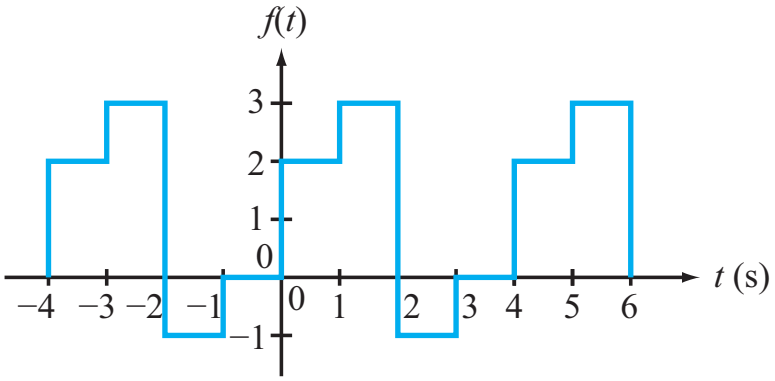


Figure E13.3

Solution:

(a)

$$f(t) \neq f(-t) \quad \Rightarrow \quad \text{no even symmetry}$$

$$f(t) \neq -f(-t) \quad \Rightarrow \quad \text{no odd symmetry}$$

(b)

$$a_0 = \frac{2 \times 1 + 3 \times 1 + (-1) \times 1}{4} = 1.$$

(c) $g(t) = [f(t) - a_0]$ has odd symmetry.

Exercise 13-4 The RL circuit shown in Fig. E13.4(a) is excited by the square-wave voltage waveform of Fig. E13.4(b). Determine $v_{out}(t)$.

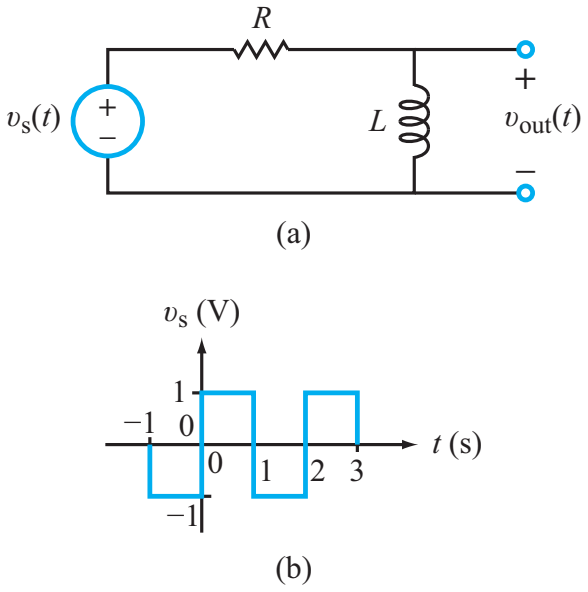


Figure E13.4

Solution: From the waveform, we deduce that

$$T = 2 \text{ s}, \qquad \omega_0 = \frac{2\pi}{T} = \pi \text{ rad/s}, \qquad A = 1 \text{ V}.$$

Step 1

From entry #2 in Table 13-2,

$$\begin{aligned} v_s(t) &= \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{4A}{n\pi} \sin\left(\frac{2\pi nt}{T}\right) \\ &= \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{4}{n\pi} \sin n\pi t \\ &= \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{4}{n\pi} \cos(n\pi t - 90^\circ) \text{ V}. \end{aligned}$$

Thus,

$$A_n = \frac{4}{n\pi}, \qquad \phi_n = -90^\circ.$$

Step 2

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_{\text{out}}(\omega)}{\mathbf{V}_{\text{s}}(\omega)} = \frac{j\omega L}{R + j\omega L}\,.$$

Step 3

With $\omega_0 = \pi$ rad/s and $\phi_n = -90^\circ$,

$$\begin{aligned}
\upsilon_{\text{out}}(t) &= a_0\,\mathbf{H}(\omega = 0) + \sum_{n=1}^{\infty} A_n\,\Re\mathfrak{e}\{\mathbf{H}(\omega = n\omega_0)\,e^{j(n\omega_0 t + \phi_n)}\} \\
&= \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{4}{n\pi}\,\Re\mathfrak{e}\left\{\frac{jn\omega_0 L}{R + jn\omega_0 L}\,e^{j(n\omega_0 t + \phi_n)}\right\} \\
&= \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{4L}{\sqrt{R^2 + n^2\pi^2 L^2}}\cos(n\pi t + \theta_n)\ \text{ V},
\end{aligned}$$

with

$$\theta_n = -\tan^{-1}\left(\frac{n\pi L}{R}\right)\,.$$

Exercise 13-5 What will the expression given by Eq. (13.43) simplify to if the associated circuit segment is (a) purely resistive or (b) purely reactive?

Solution: (a) $P_{\text{av}} = V_{\text{dc}} I_{\text{dc}} + \frac{1}{2} \sum_{n=1}^{\infty} V_n I_n$, because $\phi_{v_n} = \phi_{i_n}$, (b) $P_{\text{av}} = 0$, because for a capacitor, $I_{\text{dc}} = 0$ and $\phi_{v_n} - \phi_{i_n} = 90°$; and for an inductor, $V_{\text{dc}} = 0$ and $\phi_{v_n} - \phi_{i_n} = -90°$.

Exercise 13-6 For a single rectangular pulse of width τ, what is the spacing $\Delta\omega$ between first nulls? If τ is very wide, will its frequency spectrum be narrow and peaked or wide and gentle?

Solution: $\Delta\omega = 4\pi/\tau$. Wide τ leads to narrow spectrum.

Exercise 13-7 Use the entries in Table 13-4 to determine the Fourier transform of $u(-t)$.

Solution: From Table 13-4,

$$\text{sgn}(t) \quad \Longleftrightarrow \quad \frac{2}{j\omega},$$

$$u(t) \quad \Longleftrightarrow \quad \pi\,\delta(\omega) + \frac{1}{j\omega}.$$

Also,

$$\text{sgn}(t) = u(t) - u(-t).$$

Hence,

$$u(-t) = u(t) - \text{sgn}(t),$$

and the corresponding Fourier transform is

$$u(-t) \quad \Longleftrightarrow \quad \pi\,\delta(\omega) + \frac{1}{j\omega} - \frac{2}{j\omega}$$

$$= \pi\,\delta(\omega) - \frac{1}{j\omega}.$$

Exercise 13-8 Verify the Fourier transform expression for entry #10 in Table 13-4.

Solution:

$$\cos \omega_0 t \; f(t) = \left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \right) f(t).$$

Applying Property 5 in Table 13-5,

$$\frac{1}{2} e^{j\omega_0 t} \; f(t) \quad \Longleftrightarrow \quad \frac{1}{2} \mathbf{F}(\omega - \omega_0),$$

$$\frac{1}{2} e^{-j\omega_0 t} \; f(t) \quad \Longleftrightarrow \quad \frac{1}{2} \mathbf{F}(\omega + \omega_0).$$

Hence,

$$\cos \omega_0 t \; f(t) \quad \Longleftrightarrow \quad \frac{1}{2} [\mathbf{F}(\omega - \omega_0) + \mathbf{F}(\omega + \omega_0)].$$

Exercise 13-9 Verify the reversal property given by Eq. (13.85).

Solution: By the definition given by Eq. (13.65a),

$$\mathbf{F}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt.$$

Replacing ω with $-\omega$ gives

$$\mathbf{F}(-\omega) = \int_{-\infty}^{\infty} f(t)\, e^{j\omega t}\, dt,$$

which is the same as the complex conjugate $\mathbf{F}^*(\omega)$:

$$\mathbf{F}^*(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{j\omega t}\, dt.$$

Hence,

$$\mathbf{F}(-\omega) = \mathbf{F}^*(\omega).$$